# 香港青少年數學精英選拔賽

# The Hong Kong Mathematical High Achievers Selection Contest

# 1999-2000

## 建議題解 Suggested Solutions

Part A(甲部)

1. Let x = 654321. The expression becomes $x^2-(x+2)(x-2)=x^2-x^2+4=4$.
   設 x = 654321. 所求數值爲 $x^2-(x+2)(x-2)=x^2-x^2+4=4$.

2. $\frac{a+b}{a-b}=\frac{7}{4}\Rightarrow 4a+4b=7a-7b\Rightarrow 3a=11b\Rightarrow \frac{a}{b}=\frac{11}{3}\Rightarrow \frac{a^2}{b^2}=\frac{121}{9}$

3. From the unit digits, 12-c=3, ∴c=9. From the tens digits, 10+(b-1)-8=7, ∴b=6. From the hundreds digits, a=8-4=4. ∴a+b+c=19.
   考慮個位數, 得 12-c=3, ∴c=9. 再考慮十位數, 得 10+(b-1)-8=7, ∴b=6. 最後考慮百位數, 得 a=8-4=4. ∴a+b+c=19.

4. The left side of square 'C' is adjacent to 'A' while 'D' is adjacent to the right side of the square 'C', therefore 'D' is opposite to 'A'.
   方格' C '的左邊將與' A '相鄰，而' D '則與方格' C '的右邊相鄰，所以' A '的對面應該是' D '。

5. Let O be the intersection of AC and BD. Triangle DOC is 1/4 of the square ABCD while the shaded part is 1/2 of the DOC. Therefore the shaded part is 1/8 of the square ABCD.
   設 O 爲 AC 與 BD 的交點. 三角形 DOC 的面積 是 正方形 ABCD 的 1/4，而陰影部份是三角形 DOC 的 1/2. 所以陰映部份是正方形 ABCD 的 1/8.

6. x+y=7(x-y), ∴8y=6x
   xy=24(x-y), ∴xy=24x-24y=24x-18y=6x
   ∴x(y-6)=0; y=6 & x=8; xy=48

7. If there are n boys, the total number of students will be n+1.1n=21n/10. n must be a multiple of 10 and 21n/10≤40, so n=10 and there are 11 girls.
   設有 n 個男生, 則學生總人數爲 n+1.1n=21n/10. 因 n 爲 10 的倍數 而 21n/10≤40, 所以 n=10 而女生人數爲 11.

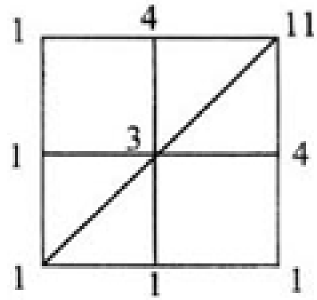
8. Let k=998. a-b=(k+1)(k×1001001+1)-k[(k+1)×1001001-1]=k+1+k=1997
   設 k=998. a-b=(k+1)(k×1001001+1)-k[(k+1)×1001001-1]=k+1+k=1997

9. 1-2-3+4+5-6-7+8+9-......+1996+1997-1998-1999
   =(1-2-3+4)+(5-6-7+8)+......+(1993-1994-1995+1996)+(1997-1998-1999)
   =1997-1998-1999= -2000

10. Let x cm be the length of the square. The perimeter of each rectangle is 2x+2x/5=24, so x=10. The radius of the circle is therefore $5\sqrt{2}$ cm and the area of the circle is $50\pi$ $cm^2$.
    若正方形邊長爲 x cm，則每個長方形的周界是 2x+2x/5=24, 所以 x=10. 因而求得圓半徑爲 $5\sqrt{2}$ cm ， 而面積爲 $50\pi$ $cm^2$.

11. Each interior angle of the octagon is 135°. BC//AD, ∴∠BAD=180-∠ABC=180°-135°=45°. Similarly, ∠HAF=45°. Hence ∠DAF=∠BAH-∠BAD-∠HAF=45°.
正八邊形內角爲 135°，而 BC//AD, ∴∠BAD=180-∠ABC=180°-135°=45°. 同樣可求得 ∠HAF=45°. 因此 ∠DAF=∠BAH-∠BAD-∠HAF=45°.

12. We may count the number of ways of getting to each point, as shown in the diagram, which is the sum of those numbers at the preceding points. There are altogether 11 possible paths.
如圖所示，計算通往每點可行路徑數目，亦即是直接通往該點的其他點上數字之和，可得總路徑數目爲 11。




13. Let the number be abcd. We know that 1200×9 is already a 5-digit number, therefore a must be 1 and b must be 0 or 1. As a=1, d must be 9. If b=1, 1109 is not the answer and 1119×9 is already a 5-digit number. So b must be 0 and the number is 1089.
設該數爲 abcd. 因 1200×9 已是一 五位數，可知 a 必是 1 而 b 只可取 0 或 1. 因 a=1, d 必是 9. 若 b=1, 可知 1109 並非 所求數而 1119×9 已是一 五位數. 因此 b 必是 0 而該數爲 1089.

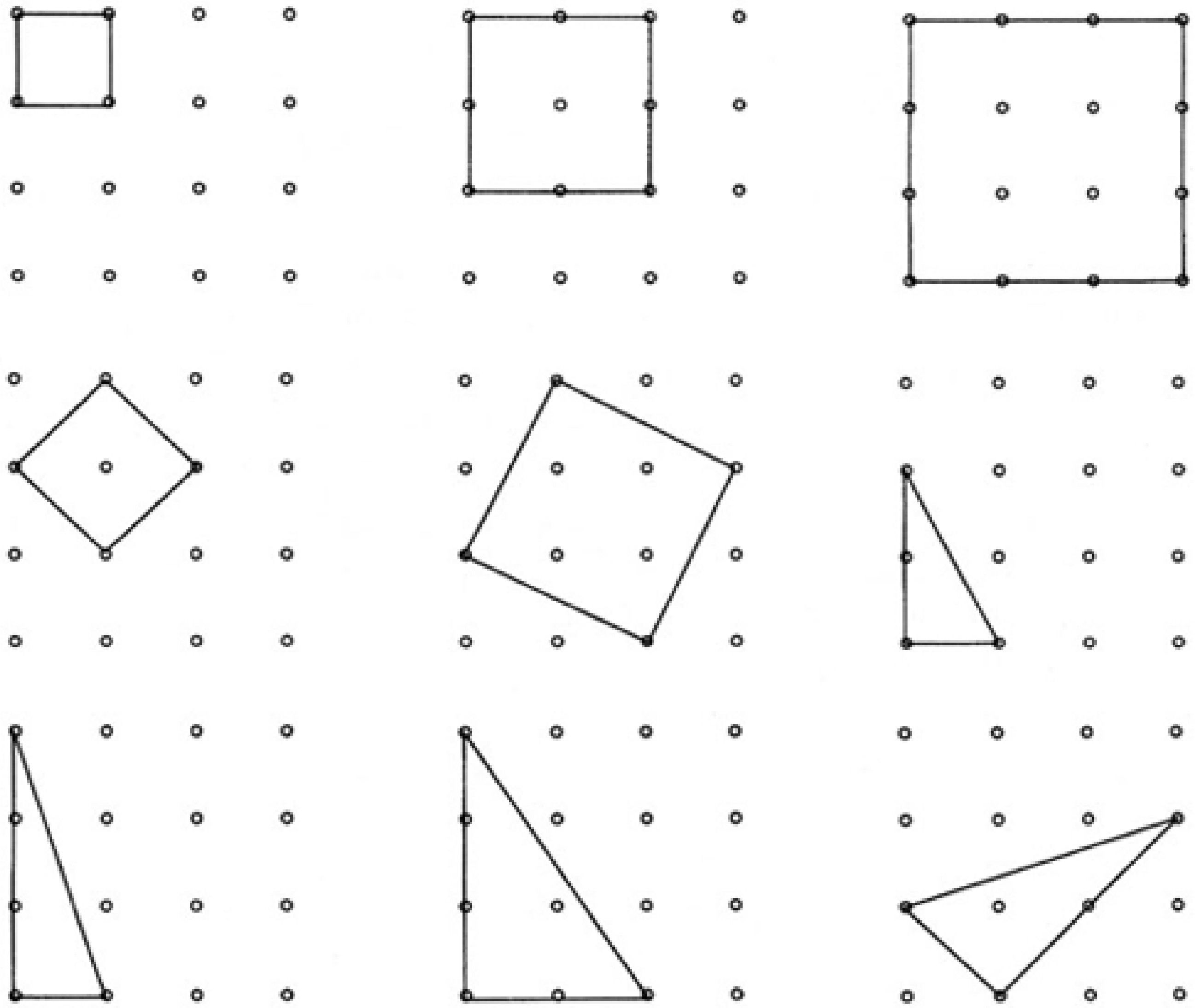
14. ∠DEC is a right angle and the area of triangle DEC = (3)(4)/2=6. Therefore the area of the rectangle is 12 and BC=12/5. Perimeter of the rectangle is therefore 14.8.
∠DEC 是一直角 而三角形 DEC 的面積 = (3)(4)/2=6. 因此長方形的面積= 12 而 BC=12/5. 由此可知長方形的周界爲 14.8.

Part B(乙部)

15. $2^{1000} = 2^{4(250)} = 16^{250}$ & $3^{750} = 3^{3(250)} = 27^{250}$

$16 < 27, \therefore 16^{250} < 27^{250}$ & $2^{1000} < 3^{750}$

16.



17.