香港青少年數學精英選拔賽

The Hong Kong Mathematical High Achievers Selection Contest

2010 – 2011

時限：兩小時

Time allowed: 2 hours

除特別指明外，數值答案應用真確值表示。

Unless otherwise specified, numerical answers should be exact.

甲部 Part A (36分 / 36 marks)

把答案填在答題紙所提供的位置。

Write the answers on the spaces provided in the answer sheet.

1. 若取至 4 位有效數字，$\overline{2010abc12011} \approx 2.011\times10^{11}$，其中三位數 $\overline{abc}$ 有多少種不同的寫法？

   If correct to 4 significant figures, $\overline{2010abc12011} \approx 2.011\times10^{11}$, how many different 3-digit numbers $\overline{abc}$ are there?

2. 若 $2011^x = 2$ 及 $2011^y = 3$，求 2011^{3x+2y} 的值。

   If $2011^x = 2$ and $2011^y = 3$, find the value of 2011^{3x+2y}.

3. 解方程 $\frac{x}{11\times13}+\frac{x}{13\times15}+\cdots+\frac{x}{2009\times2011}=\frac{11}{2011}$。

   Solve $\frac{x}{11\times13}+\frac{x}{13\times15}+\cdots+\frac{x}{2009\times2011}=\frac{11}{2011}$.

4. 若 $A-B=2009$，$B-C=-2010$ 及 $C-D=2011$，求 $\frac{(A-D)}{(A-C)(B-D)}$ 的值。

   If $A-B=2009$, $B-C=-2010$ and $C-D=2011$, find the value of $\frac{(A-D)}{(A-C)(B-D)}$.

5. 將 2011 寫成兩正整數平方之差。

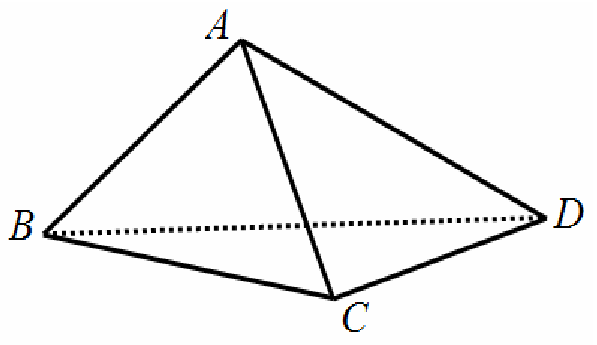
   Write 2011 as a difference of two squares of positive integers.

6. How many distinct numbers are there in the 4 × 4 × 4 "multiplication cube", that is, the sixty-four numbers A × B × C where *A*, *B*, *C* are integers running from 1 to 4 ?

   在 4 × 4 × 4 的「立方積」 (即: A × B × C，其中 A、B 和 C 爲整數 1 至 4 的 64 個數) 中有多少個不同的數字？

7. 如圖，$ABCD$ 是一個三角錐體，其中$\angle BAC = \angle CAD = \angle BAD = 90^{o}$，

且 $AB = 6$ cm、$AC = 7$ cm 及 $AD = 8$ cm，求該三角錐體 $ABCD$ 的體積。



In the figure, $ABCD$ is a triangular pyramid with $\angle BAC = \angle CAD = \angle BAD = 90^{o}$，$AB = 6$ cm、$AC = 7$ cm and $AD = 8$ cm. Find the volume of the pyramid $ABCD$.

8. 若 S = 1 + 11 + 111 + 1 111 + …+ 1 111 …111 111 (2011 項相加)，S 的最後 5 個數字是甚麼？

If S = 1 + 11 + 111 + 1 111 + …+ 1 111 …111 111 (sum of 2011 terms), what are the last five digits of S?

9. 若 k 個連續正整數之和爲 2010，求 k 的最大可能值。

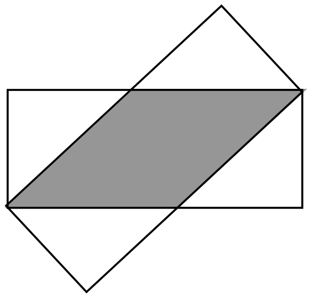
If the sum of k consecutive positive integers is 2010, find the greatest possible value of k.

10. 將若干個邊長爲 1cm 的小立方體構成一個邊長爲 4cm 的大立方體，其中 x 個小立方體恰好有四個面與其他小立方體的面相連接，求 x 的值。

A cube with sides of length 4 cm is made up of a number of smaller cubes with sides of length 1 cm. Among these smaller cubes, x of them are adjacent to exactly 4 other smaller cubes. Find the value of x.

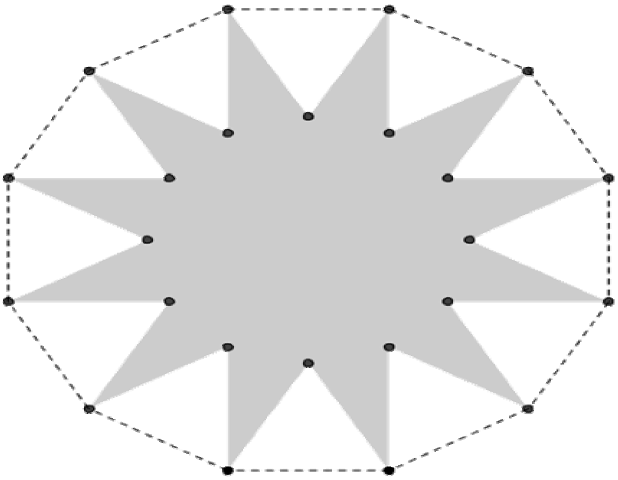
11. 如圖，兩個全等的 3 cm × 7 cm 的長方形重疊在一起，求重疊部份的面積。

In the figure, two identical 3 cm × 7 cm rectangles are overlapping each other. Find the area of the overlapping region.



12. 如圖，從一個邊長 2cm 的正十二邊形內移去十二個等邊三角形後，得以下星形。求該星形的面積。

In the figure, the star-shaped figure is formed by removing 12 equilateral triangles from a regular dodecagon of side length 2 cm. What is the area of the star-shaped figure?



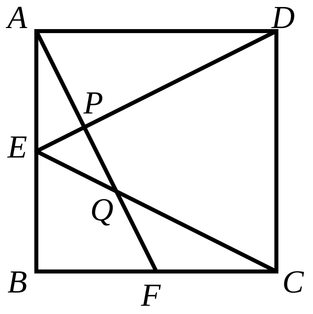
13. 某中一級補習班中，有 15 名學生修讀數學科，12 名學生修讀中文科，9 名學生修讀英文科。 已知該補習班共有 22 人，且每人至少修讀以上其中一科，那麼最多有多少名學生修讀了全部三科？

In a certain S1 tutorial class, 15 students take Mathematics, 12 take Chinese and 9 take English. It is known that there are 22 students in the class and each student must take at least one of the above subjects. At most how many students take all the three subjects?

14. 設直角三角形的兩條直角邊長分別爲 a、b，斜邊長爲 c。若 a、b、c 均爲整數，且 $c=\frac{1}{3}ab-(a+b)$，求 c 的最大值。

Given that the length of the hypotenuse of a right-angled triangle is c and the length of the other two sides are a and b. If a, b, c are integers and $c=\frac{1}{3}ab-(a+b)$, find the greatest value of c.

15. 圖中 *ABCD* 爲一正方形，*E* 及 *F* 分別爲 *AB* 及 *BC* 的中點，*AF* 分別交 *ED* 及 *EC* 於 *P* 及 *Q*。若 *AB* = 1，求ΔEPQ 的面積。



The figure shows a square *ABCD*. *E* and *F* are mid-points of *AB* and *BC* respectively. *AF* intersects *ED* and *EC* at *P* and *Q* respectively. If *AB* = 1, find the area of ΔEPQ.

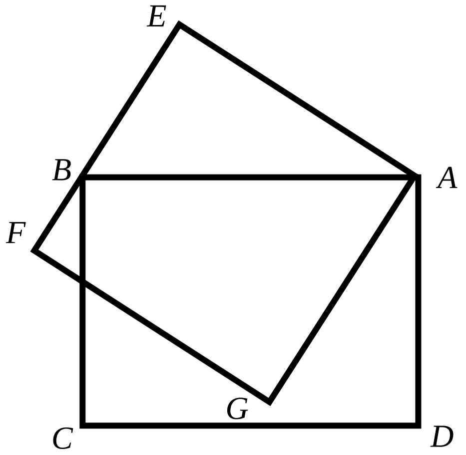
16. 求 $\frac{1}{2^{10}+1}+\frac{1}{2^{10}+2}+\frac{1}{2^{10}+2^2}+\frac{1}{2^{10}+2^3}+\cdots+\frac{1}{2^{10}+2^{20}}$ 的値。

Find the value of $\frac{1}{2^{10}+1}+\frac{1}{2^{10}+2}+\frac{1}{2^{10}+2^2}+\frac{1}{2^{10}+2^3}+\cdots+\frac{1}{2^{10}+2^{20}}$.

17. 求 $\sqrt{2011+\sqrt{2011+\sqrt{\cdots+\sqrt{2011}}}}$ (2011 共出現了 2011 次) 的整數部份。

Find the integral part of $\sqrt{2011+\sqrt{2011+\sqrt{\cdots+\sqrt{2011}}}}$ (the number of 2011 is 2011.)

18. 圖中 *ABCD* 及 *AEFG* 皆爲正方形，且 *B* 在 *EF* 上。若 *AE* = 4 及 *AB* = 5，求 *FC* 之長度。



The figure shows 2 squares *ABCD* and *AEFG*. *B* lies on *EF*. If *AE* = 4 and *AB* = 5. Find the length of *FC*.

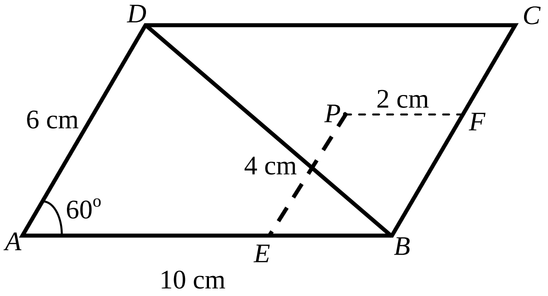
乙部 Part B (18分 / 18 marks)

把完整的題解和答案寫在答題紙所提供的位置。

Answer the following questions with full solutions on the spaces provided in the answer sheet.

19. *ABCD* 為一平行四邊形，其中 $AB = 10$ cm、 $AD = 6$ cm 及 $\angle BAD = 60^o$。P 是在 ΔBCD 內的一點。點 E 和 F 分別為在 AB 和 BC 上使得 $PE//BC$ 和 $PF//AB$ 。若 $PE = 4$ cm 及 $PF = 2$ cm，求 ΔBDP 的面積。 (可將答案以根式表示。)

ABCD is a parallelogram with $AB = 10$ cm, $AD = 6$ cm and $\angle BAD = 60^o$. P is a point inside ΔBCD. E and F are points on AB and BC respectively such that $PE//BC$ and $PF//AB$. If $PE = 4$ cm and $PF = 2$ cm. Find the area of ΔBDP. (You can leave the answer in surd form.)



20. 將 1、2、3、4、…、2010、2011 等 (2011 個整數) 寫在黑板，問至少要擦去多少個整數使得留在黑板上的全部整數的乘積的末位數字是 2？

Write 1, 2, 3, 4, … , 2010, 2011 (2011 integers) on the blackboard, what is the minimum number of integers should be removed from blackboard, so that the last digit of the product of the remaining numbers is 2 ?

21. 試找出一個 8 位數，它的各數位數字均是 1 或 2，且能被 256 整除。

(提示 : $256 = 2^8$.)

Find an 8-digit number with 1 or 2 as digits such that it is divisible by 256.
(Hint : $256 = 2^8$.)

擬題委員會： 蕭文強教授(香港大學)、吳端偉助理教授(香港大學) 、李文生先生(香港大學)、馮德華老師、郭家強老師、潘維凱老師、徐崑玉老師、徐鳳鳴老師