<center>

香港青少年數學精英選拔賽

The Hong Kong Mathematical High Achievers Selection Contest

2008 – 2009

</center>

時限：兩小時

Time allowed: 2 hours

除特別指明外，數值答案應用真確值表示。

Unless otherwise specified, numerical answers should be exact.

甲部 Part A

把答案填在答題紙所提供的位置。

Write the answers on the spaces provided in the answer sheet.

1. 圖中長方形 ABCD 是由 7 個大小相同的小長方形所組成，而長方形 ABCD 的周界是 136cm。求長方形 ABCD 的面積。

   In the figure, rectangle ABCD is formed by 7 smaller identical rectangles. The perimeter of the rectangle ABCD is 136 cm. Find the area of the rectangle ABCD.
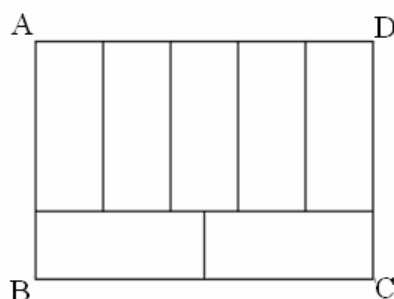

2. 圖中兩條闊度皆為 1cm 的紙條，交叉重疊放在一起，且它們的夾角為 60°，求圖中陰影部分的面積。

   In the figure, two paper strips with a width of 1cm cross over each other. The angle between these two strips is 60°. Find the area of the shaded region.
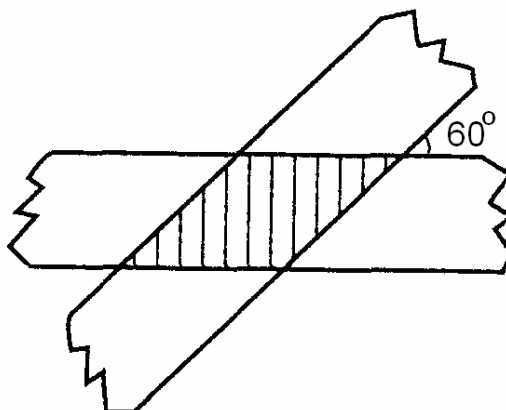
3. 圖中 ABC 是一個銳角三角形，M 是 BC 上的中點，AE 的長度是 BE 的 2 倍，若ΔAFC 的面積是 6，求ΔAEF 的面積。

In the figure, ABC is an acute angled triangle.   M is the mid-point of BC.   The length of AE is twice that of BE.   If the area of ΔAFC is 6, find the area of ΔAEF.
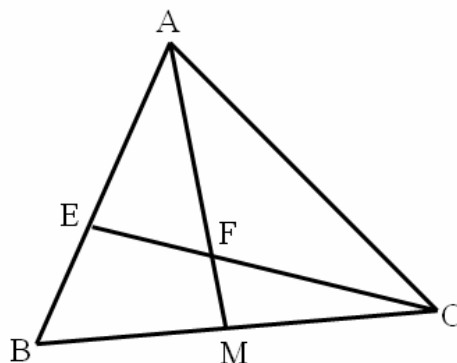
4. 袋中有 2009 個大小相同的彩色球，其中紅色球有 1000 個，黃色球有 801 個及其餘都是藍色球。從袋中至少抽出多少個球，才能保証這些球中至少有 801 個是同色的？

There are 2009 identical colour balls, of which 1000 are red, 801 are yellow and the rest are blue. In order to get at least 801 balls of the same colour, what is the minimum number of balls that must be drawn?

5. 計算 $(\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\Lambda+\frac{1}{2009})(1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\Lambda+\frac{1}{2008})$

$$-(1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\Lambda+\frac{1}{2009})(\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\Lambda+\frac{1}{2008})。$$

Calculate $(\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\Lambda+\frac{1}{2009})(1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\Lambda+\frac{1}{2008})$

$$-(1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\Lambda+\frac{1}{2009})(\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\Lambda+\frac{1}{2008}).$$

6. 已知 $x+\frac{1}{x}=3$，求 $x^4+3x^3-16x^2+3x+2009$ 的值。

It is given that $x+\frac{1}{x}=3$, find the value of $x^4+3x^3-16x^2+3x+2009$.

7.　二十三名學生在一次測驗中共得 2009 分，每人得分互不相同，且其中最高得分是 99 分及最低得分是 A 分。求 A 的最小值。

The total score of twenty-three students in a test is 2009 and no two students have the same score. Among them, the highest score obtained is 99 and the lowest score is A. Find the smallest value of A.

8.　若 B 爲一個六位數 $\overline{a2009b}$，且能被 12 整除，求 B 的最大值。

If B is a 6-digit number $\overline{a2009b}$ and is divisible by 12, find the greatest value of B.

9.　已知 $(5x+1)^{2008} + \sqrt{(y-5)} = 0$，求 $x^{2009} \times y^{2010}$ 的值。

It is given that $(5x+1)^{2008} + \sqrt{(y-5)} = 0$. Find the value of $x^{2009} \times y^{2010}$.

10.　若 x 爲一正整數，且 x^2 爲 11 個連續整數的平方和，求 x 的最小值。

If x is a positive integer and x^2 is the sum of the squares of 11 consecutive integers, find the smallest value of x.

11.　將若干枝鉛筆分給學生。
若分給 A、B 及 C 班學生，每人分得 42 枝鉛筆；
若只分給 A 班學生，每人分得 105 枝鉛筆；
若只分給 B 班學生，每人分得 98 枝鉛筆；
若只分給 C 班學生，每人分得多少枝鉛筆？

Divide a certain number of pencils among students.
If the pencils are distributed to classes A, B and C, each will get 42 pencils;
if distributed to class A only, each will get 105;
if distributed to class B only, each will get 98.
If the pencils are distributed to class C only, how many pencils can each get?

12. 今年小明爺爺的年齡是一個兩位數，將這兩位數的數字交換得到的數字就是小明爸爸的年齡，且爺爺的年齡與爸爸的年齡的差恰好是小明年齡的 4 倍，求今年小明的年齡。

The age of Tim's grandpa is a two-digit number. When the two digits are reversed, the age of his father is obtained. If the difference between the ages of his grandpa and father is 4 times Tim's age, find Tim's present age.

13. 將水注入一個半徑為 7cm 的圓柱體容器中，使得水高 8cm，再將一個長 6cm、闊 9cm 及高 15.4cm 的長方體形鐵塊放入這圓柱體容器，並使它直立在這圓柱體容器中。假設水沒有溢出，問水位升高了多少 cm？(取 $\pi = \dfrac{22}{7}$)

Pour water into a cylinder with a radius of 7cm until the depth of water is 8cm high. Then a rectangular iron bar with length 6cm, width 9cm and height 15.4cm is put into the cylinder so that it is vertically standing inside the cylinder. Suppose no water is leaked, what is the rise in water level in cm? (Take $\pi = \dfrac{22}{7}$)

14. 求方程 $(x^2 - x - 1)^{x+2009} = 1$ 的所有根的和。

Find the sum of all roots of the equation $(x^2 - x - 1)^{x+2009} = 1$.

15. 圖中 ABCD 和 AFGE 是相似的長方形，BD 將長方形 AFGE 分為兩部份，這兩部份的面積分別是 1cm^2 和 49cm^2。求長方形 ABCD 的面積。

In the figure, ABCD and AFGE are similar rectangles. BD cuts AFGE into two parts with areas 1cm^2 and 49cm^2 respectively. Find the area of rectangle ABCD.
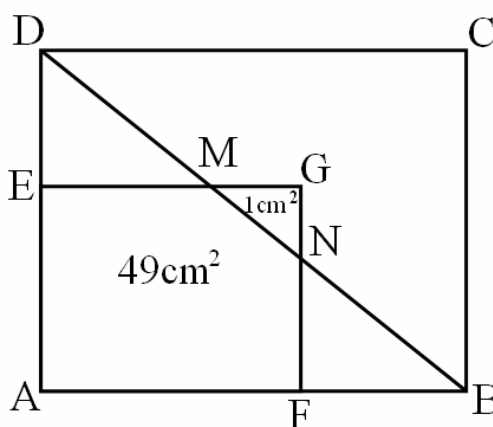
16. 在數列 $1^1, 2^2, 3^3, 4^4, \Lambda, 2008^{2008}, 2009^{2009}$ 中，設最小的 1005 項之和的末位數字爲 M，及最大的 1004 項之和的末位數字爲 N，求 $M - N$ 的值。

In the sequence $1^1, 2^2, 3^3, 4^4, \Lambda, 2008^{2008}, 2009^{2009}$, let the last digit of the sum of the smallest 1005 terms be M and the last digit of the sum of the greatest 1004 terms be N. Find the value of $M - N$.

17. 2009 年 2 月 7 日是星期六。問下一個使 2 月 7 日是星期六的是那一個年份？

February 7 is a Saturday in 2009. What is the next year nearest to 2009 such that February 7 again falls on a Saturday?

18. 下圖中，半圓 ACB 的直徑爲 2cm。以 O_1 及 O_2 爲圓心的兩圓有相同的半徑。求以 O_3 爲圓心的圓的半徑。(答案可以根式表示。)

In the figure, the diameter of the semicircle ACB is 2cm. The circles centered at O_1 and O_2 have the same radii. Find the radius of the circle centered at O_3. [You can leave the answer in surd form.]
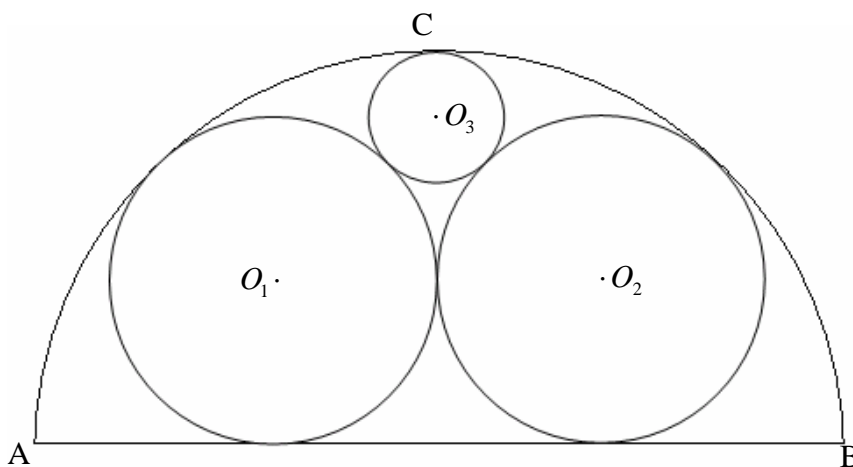
- 甲部完 -

乙部 Part B

把完整的題解和答案寫在答題紙所提供的位置。

Answer the following questions with full solutions on the spaces provided in the answer sheet.

19. 在一象棋比賽中，10 名參賽者被分成若干隊，每一隊的每一參賽者都要與其他隊的每一參賽者進行一場比賽，而同一隊的參賽者之間則不進行比賽。已知這次比賽最後共進行了 31 場比賽，那麼在這次比賽中，該 10 名參賽者被分成多少隊？

   In a chess competition, 10 players are divided into a certain number of teams. Each player of a given team plays one game with each player of all the other teams, while players within the same team do not play among themselves. It is known that 31 games have been played at the end of this chess competition. What is the number of teams in this competition?

20. 使用 1, 2, 3, 4, 5, 6, 7, 8, 9 全部 9 個數字組成一個 9 位數，使由該數由左至右數起的頭 n 位數組成的數字可被 n 整除。(提示 : 你可先找出中間三個數的可能數字。)
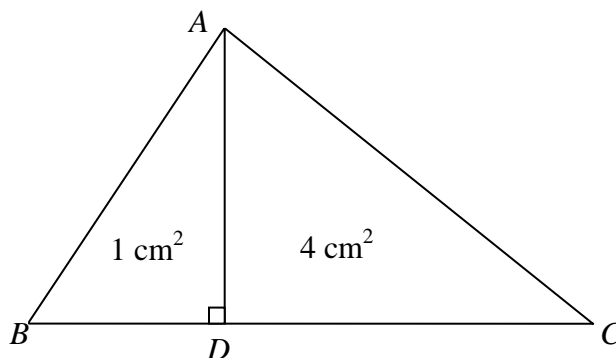
   Use ALL the nine digits 1, 2, 3, 4, 5, 6, 7, 8, 9 to form a nine-digit number such that the number formed by the first n digits, counting from the left to the right, is divisible by n.

   [Hint : You may find out the possible three digits in the middle portion of the number first.]

21. (a) 下圖中，兩直角三角形 ABD 及 ADC 的面積分別爲 1 cm^2 及 4 cm^2。若三角形 ABC 爲鈍角三角形，求 BD 的範圍值。

   (b) 求一個由兩個面積分別爲 1 cm^2 及 4 cm^2 的直角三角形所組成的直角三角形 ABC 的三邊。(答案可以根式表示。)

   (a) In the figure, the areas of the right-angled triangles ABD and ADC are 1 cm^2 and 4 cm^2 respectively. If ABC is an obtuse triangle, find the range of values of BD.

   (b) Find the length of the three sides of a right-angled triangle composed of two right-angled triangles of area equal to 1 cm^2 and 4 cm^2 respectively? [You can leave your answer in surd form.]

- 乙部完 -