香港青少年數學精英選拔賽

The Hong Kong Mathematical High Achievers Selection Contest

2007 – 2008

時限：兩小時

Time allowed: 2 hours

除特別指明外，數值答案應用真確值表示。

Unless otherwise specified, numerical answers should be exact.

甲部Part A

把答案填在答題紙所提供的位置。

Write the answers on the spaces provided in the answer sheet.

1. Find the number of reduced proper fractions with denominator equals 143.

   以 143 作為分母的最簡真分數有多少個？

2. Evaluate $\dfrac{\left(2008^2-2008-6\right)\left(2008^2+2\times2008-3\right)}{2005\times2007\times2009\times2010\times2011}$

   計算 $\dfrac{\left(2008^2-2008-6\right)\left(2008^2+2\times2008-3\right)}{2005\times2007\times2009\times2010\times2011}$ 的值。

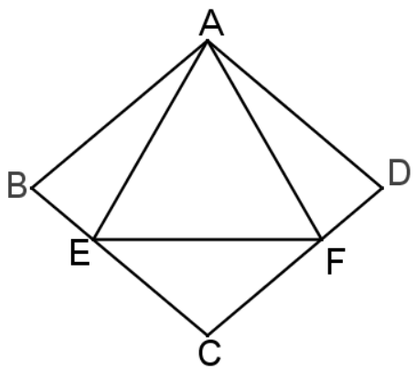
3. Let $a = 3^{50}$, $b = 15^{25}$, $c = 33^{20}$ and $d = 1024^{10}$. Arrange a, b, c and d in decreasing order of magnitude.

   設 $a = 3^{50}$、$b = 15^{25}$、$c = 33^{20}$ 和 $d = 1024^{10}$。將 a、b、c 和 d 由大至小排列。

4. A rectangle, whose sides are of integer length, has its perimeter and area equal in value. Find the area(s) of all these possible rectangle(s).

   若長方形的邊長為整數，且它的周界與面積的值相等，求所有滿足以上條件的長方形的面積。

5. The side length of an equilateral triangle AEF and that of a rhombus ABCD are equal with point E and F lying on BC and CD respectively. Find the size of $\angle ABE$.



   等邊三角形 ΔAEF 的邊長與菱形 ABCD 的邊長相同，而點 E 和 F 分別在 BC 和 CD 上，求 $\angle ABE$ 的大小。

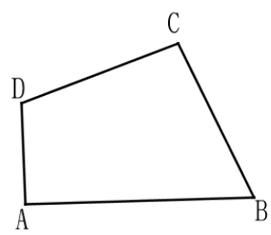
6. In an isosceles triangle ABC, D is the mid-point of BC, where AD divides the perimeter of triangle ABC into two parts of lengths 24 cm and 30 cm respectively. Find the length of AB.

等腰三角形 ABC 中，D 是 BC 的中點，且 AD 把三角形 ABC 的周界分為兩部分，其長度分別是 24cm 及 30cm。求 AB 的長度。

7. Suppose $a_{n+1} = 1 - \frac{1}{a_n}$ and $a_2 = 7$. Find a_{2008}.

設 $a_{n+1} = 1 - \frac{1}{a_n}$ 及 $a_2 = 7$。求 a_{2008}。

8. In a 4-sided convex polygon, AB = 8, CD = 3, ∠A=∠C=90° and ∠D=135°. Find area of the 4-sided convex polygon.



凸四邊形 ABCD 中，設 AB = 8、CD = 3、∠A=∠C=90°及∠D=135°。求這四邊形的面積。

9. A little child asked his teacher how old he was. His teacher said that his age was the sum of digits of his year of birth. Then, how old is his teacher now?

小孩問他的老師今年多少歲，他的老師便答道：「我今年的歲數是我出生的年份的各數字之和。」那麼他的老師今年多少歲？

10. When $\frac{2}{7}$ is expressed as a recurring decimal, a part of n digits are taken after the decimal point from the recurring decimal. If the sum of these n digits is 2008, find the value of n.

將 $\frac{2}{7}$ 化成一個循環小數後，從這個小數的小數点後截取連續的 n 個數字。若這 n 個數字之和為 2008，求 n 的值。

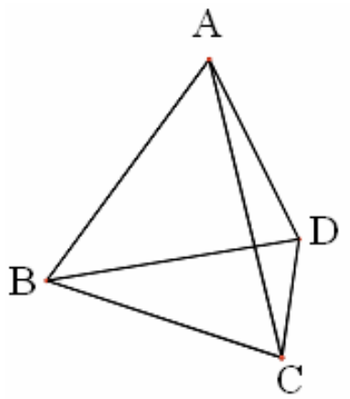
11. Find the smallest integer which is greater than $\left(\sqrt{7}+\sqrt{6}\right)^2$.

求大於 $\left(\sqrt{7}+\sqrt{6}\right)^2$ 的最小整數值。

12. Suppose r is the decimal part of $\sqrt{5}$, find the value of $r^2 + 2r + 2\sqrt{5}$.

設 r 為 $\sqrt{5}$ 的小數部分，求 $r^2 + 2r + 2\sqrt{5}$ 的值。

13. ABCD is a regular tetrahedron. The length of each edge is 1. Point M and N lie on AB and CD respectively. Find the length of the shortest distance between M and N.

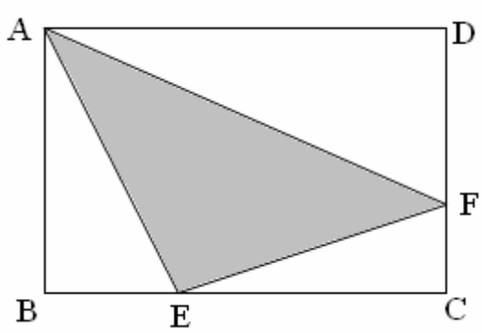


正四面體 ABCD 的斜稜長 1，而點 M 和點 N 分別在斜稜 AB 和 CD 上。求 M 和 N 之間的最短的距離。

14. The length of a cube is 3 units. It is formed by 27 smaller cubes with same size but different colours. How many different rectangular blocks can be found from this cube? (Your answer should include the number of cubes found.)

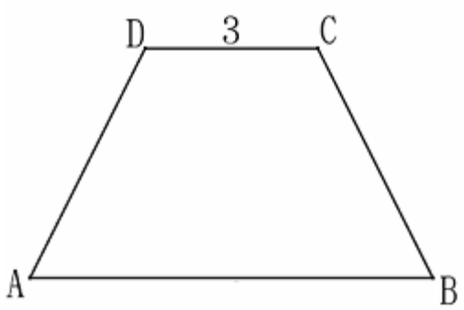
一個正方體邊長為 3 單位，它是由 27 粒大小一樣但顏色不相同的小正方體所組成。問從這個正方體中可取出多少個不同的長方體？(答案須包括正方體的數量。)

15. The area of a rectangle ABCD is 35 cm^2. The area of △ABE and that of △ADF are 5 cm^2 and 7 cm^2 respectively. Find the area of △AEF.



長方形 ABCD 的面積為 35 cm^2，△ABE 和△ADF 的面積分別為 5 cm^2 和 7 cm^2，求 ΔAEF 的面積。

16. In the diagram, ABCD is an isosceles trapezium. AB is equal to the diagonal of the trapezium. CD is equal to the height of the trapezium and its length is 3 units. Find the length of AB in the figure.



圖中 ABCD 為一等腰梯形，而 AB 的長度與梯形的對角線的長度相等，且 CD 的長度等於梯形的高，若 CD 長 3 單位，求圖中 AB 的長度。

17. 20 students chose to study physics, chemistry or mathematics. Each student must select at least two subjects. There were 12 students in physics, 15 students in chemistry and 18 students in mathematics. How many students studied all three subjects?

20 名學生選修物理、化學和數學，每名學生必須至少選兩科。物理班有 12 人，化學班有 15 人，數學班有 18 人。問共有多少人同時選修三科？

18. A and B played the game “paper, scissors and rock”. There was no draw. A showed paper 3 times, scissors 2 times, rock 1 time; B showed paper 1 time, scissors 2 times, rock 3 times. How many times did A win? [The answer may not be unique. You should give **ALL** possible, and **ONLY THOSE** possible, answers.]

甲和乙玩「包、剪、錘」的遊戲，沒有和局。甲出了 3 次包，2 次剪和 1 次錘。乙出了 1 次包，2 次剪和 3 次錘。問甲勝了幾局？[答案可能不是唯一的，你需要寫下**全部**可能，亦**僅是那些可能**發生的情況。]

- 甲部完 -

乙部 Part B

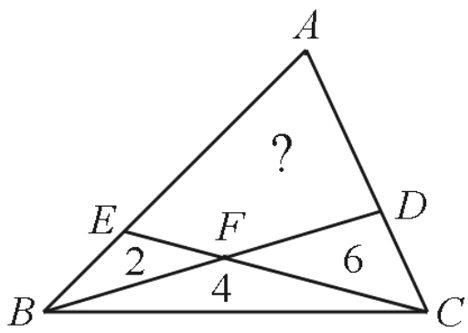
把完整的題解和答案寫在答題紙所提供的位置。

Answer the following questions with full solutions on the spaces provided in the answer sheet.

19. If an positive integer can be expressed as the difference between the squares of two positive integers, it is called a ‘wise integer’. Starting from 1 to 2008, how many ‘wise integers’ are there?

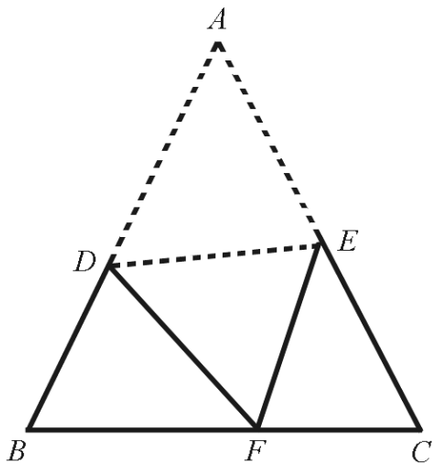
一個正整數若能以兩個正整數之平方差表示，則這個整數便稱為「智慧數」。問從 1 至 2008 中，有多少個「智慧數」？

20. D and E are respectively points on the sides AC and AB of a triangle ABC. The straight lines BD and CE intersect at F. The triangles EBF, FBC and FCD have areas equal to 2 sq. cm., 4 sq. cm. and 6 sq. cm. respectively. Find the area of the quadrilateral AEFD.



三角形 *ABC* 的邊 *AC* 和 *AB* 上各有點 *D* 和 *E*， 直線 *BD* 和 *CE* 相交於 *F* 。 三角形 *EBF*、*FBC* 和 *FCD* 的面積分別為 2 平方厘米、4 平方厘米和 6 平方厘米，求四邊形 *AEFD* 的面積。

21. ABC is an equilateral triangle. D and E are points on the sides AB and AC respectively. Suppose we choose D and E carefully so that when the triangle is folded along DE, the apex A touches BC at the point F (see the figure below). If the length of AD and AE are 91cm and 65cm respectively, find the length of BC.



ABC 為一等邊三角形，而 *D* 和 *E* 分別為 *AB* 和 *AC* 上的點。只要適當地選取點 *D* 和 *E*，並將三角形沿 *DE* 對摺，頂點 *A* 剛好落在 *BC* 上的點 *F* 上。如果 *AD* 和 *AE* 分別長 91cm 和 65cm，求 *BC* 的長度。