時限：兩小時

Time allowed: 2 hours

除特別指明外，數值答案應用真確值表示。

Unless otherwise specified, numerical answers should be exact.

甲部 Part A

把答案填在答題紙所提供的位置。

Write the answers on the spaces provided in the answer sheet.

1. 有兩個具有時針和分針的時鐘(即，非跳字時鐘)。其中一個鐘每小時慢 30 分鐘，而另一個鐘則每小時快 20 分鐘。此兩鐘現同時正確地設定於午夜，問當下一次兩鐘所示時間相同時，兩鐘所顯示的時間是甚麼?

    There are two clocks with an hour hand and a minute hand (that is, not digital clocks).   One runs 30 minutes slower each hour, and the other runs 20 minutes faster each hour.   The two clocks are set right at midnight.   What time is shown on the clocks when both look the same the next time?

2. 創作一個 4×4 方陣，使 (i) 每行、(ii) 每列、(iii) 兩條對角線 及 (iv) 在每個角落的 2×2 方陣均填滿由 1, 2, 3, 4 這 4 個數字。

    Construct a 4×4 grid of digits chosen from 1,2,3,4 such that all four digits appear in (i) each of the four rows, (ii) each of the four columns, (iii) each of the two diagonals, (iv) each of the four corner 2×2 grids.

3. 求 $\dfrac{\left(2^2 + 4^2 + ... + 2006^2\right) - \left(1^2 + 3^2 + 5^2 + ... + 2005^2\right)}{(2 + 4 + ... + 2006) - (1 + 3 + 5 + ... + 2005)}$ 。

    Calculate $\dfrac{\left(2^2 + 4^2 + ... + 2006^2\right) - \left(1^2 + 3^2 + 5^2 + ... + 2005^2\right)}{(2 + 4 + ... + 2006) - (1 + 3 + 5 + ... + 2005)}$.

4. 設 $x_1, x_2, ..., x_{10}$ 都是正整數，$x_1 < x_2 < x_3 < ... < x_{10}$，且 $x_1 + x_2 + ... + x_{10} = 2006$，求 x_5 的最大可能值。

    Given that $x_1, x_2, ..., x_{10}$ are positive integers such that $x_1 < x_2 < x_3 < ... < x_{10}$ and $x_1 + x_2 + ... + x_{10} = 2006$, find the greatest possible value of x_5 .

5. 已知一個七位數 $\overline{62xy427}$ 是 99 的倍數，求 $500x + 250y + 6$ 的值。

    It is known that a 7-digit number $\overline{62xy427}$ is a multiple of 99, find $500x + 250y + 6$.

6. 若 $n^3 + 3n + 2006$ 能被 $n + 3$ 整除而 n 為百位整數，求 n 的最大值。

    If $n + 3$ divides $n^3 + 3n + 2006$ and n is a 3-digit integer，find the greatest possible value of n.

7. 若 $[x] =$ 少於或等如 x 的最大整數。

    例如：$[4.23] = 4$； $[0.87] = 0$； $[\sqrt{3}] = 1$； $[\pi] = 3$。
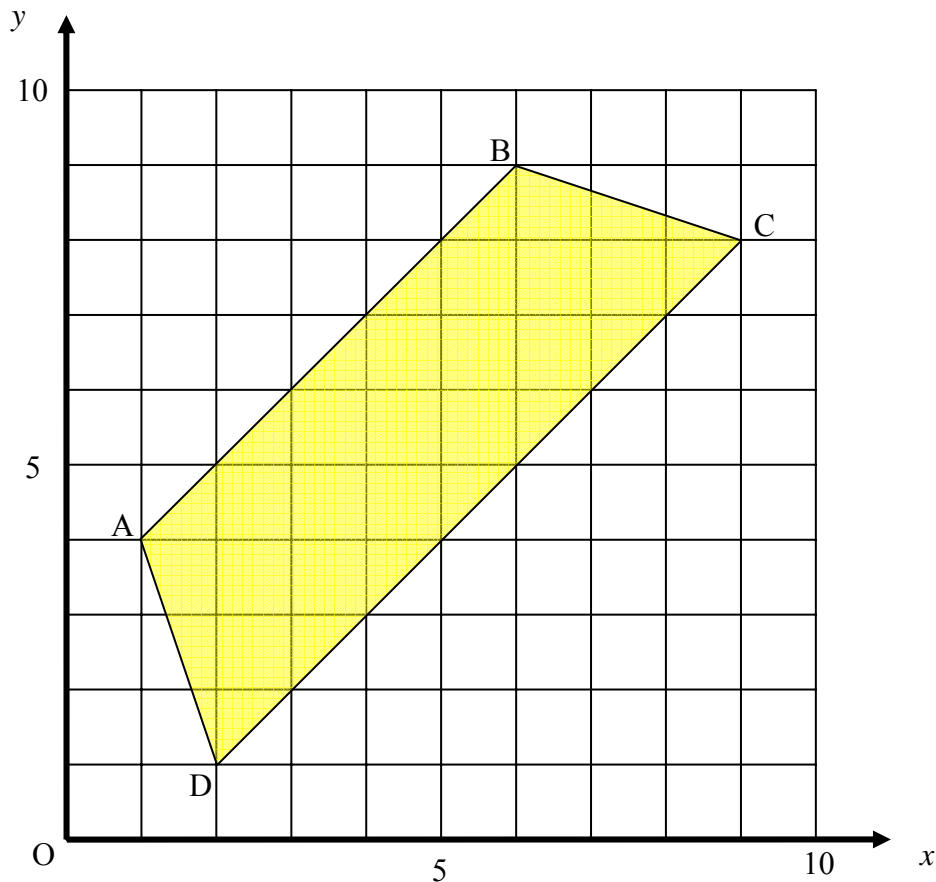
    求 x 滿足於 $x^3 - [x] = 4$ 的值。

    Let [x] = the greatest integer smaller than or equal to x.

    E.g. : $[4.23] = 4$； $[0.87] = 0$； $[\sqrt{3}] = 1$； $[\pi] = 3$。

    Find x such that $x^3 - [x] = 4$.

8. 下圖所示為一等腰梯形 ABCD。求 E 點的坐標使 ABCE 為另一等腰梯形。

   In the following figure, ABCD is an isosceles trapezium. Find the coordinates of the point E so that ABCE is another isosceles trapezium.

9. 右圖中的正方形被 AE 分成兩部分。在沒有重疊的情況下將此兩部分以三種不同方法組成一個四邊形，求這些四邊形的最大可能周界。

   The square in the diagram is cut into two pieces along AE.The two pieces are then regrouped, without overlapping, to form another quadrilateral in three different ways. Find the greatest possible perimeter of each of these new quadrilaterals.
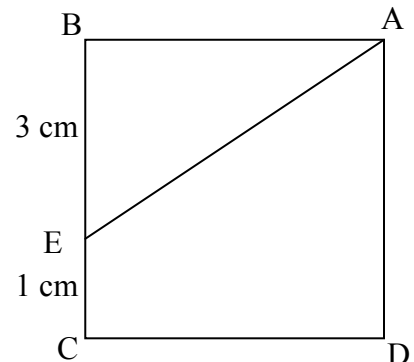
10. 兩個數的和是 2 而積亦是 2，求它們的立方的和。

    The sum and product of two numbers are both equal to 2. Find the sum of the cubes of these two numbers.

11. 當使用相同的引擎輸出時，某小輪順流航行 60km 比回程需時少 5 小時。如果該小輪把引擎輸出加倍即把靜水速度加倍，順流航行 60km 比逆流需時少 1 小時。求水流以 km/h 計的速度。

    With the same engine power, if a ferry is running for 60km with the current, it takes 5 hours fewer than when it returns. When the engine power is doubled, that means the speed of ferry in still water is doubled, it takes 1 hour fewer when the ferry is running for 60 km with the current than it is against the current. Find the speed of the current in km/h.

12. 求方程 $3x + 4y = 501$ 的正整數序偶解的數目。

    Find the number of positive integral solution(s) of the equation $3x + 4y = 501$.

13. 以 $a + b\sqrt{c}$ 的形式表 $\left(\sqrt{4 + \sqrt{4 + \sqrt{4}}}\right)^4$，而 a、b 和 c 為整數。

    Express $\left(\sqrt{4 + \sqrt{4 + \sqrt{4}}}\right)^4$ in the form of $a + b\sqrt{c}$ where a, b, c are integers.

14. ABCD 爲正方形，P 和 Q 分別爲 BC 及 DA 的中點，而陰影部分爲 AP 及 CQ 之間的一個最大的長方形。求陰影部分面積佔整個正方形 ABCD 的百分比。

    ABCD is a square and P, Q are the mid-points of BC and DA. If the shaded region is the largest rectangle lying between AP and CQ. Find the proportion of the shaded area to the square ABCD.
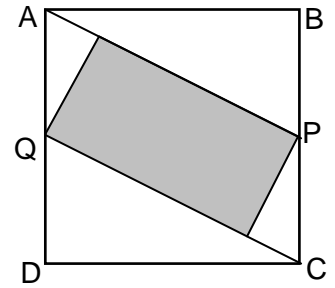
15. ΔABC 中，從頂點 A, B 和 C 各畫一線至對邊，將三角形分成 6 份，其中四個部份之面積如圖所示。求ΔABC 的面積。

    In ΔABC, lines are drawn from vertices A, B and C to their opposite sides so that the triangle is divided into 6 parts.
    The areas of some parts are shown in the figure. Find the area of ΔABC.
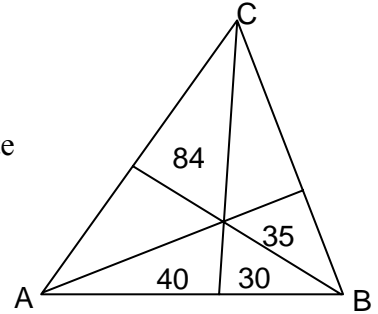
16. ABP 是一等邊三角形，其中 AB 長 1 cm，AXYZ 是一邊長爲 2 cm 的正方形。ABP 位於 AXYZ 之外而 B 點於 AX 上。若三角形以 B、P 如此類推的點爲支點按逆時針方向沿正方形的邊旋轉，直至 P、A 和 B 點返回原位。求 P 點移動路線的長度(cm)。

    Equilateral triangle ABP with side AB of length 1 cm is placed outside square AXYZ with side of length 2 cm so that B is on side AX. The triangle is rotated anti-clockwise about B, then P, and so on along the sides of the square until P, A and B all return to their original positions. Find the length of the path in cm travelled by vertex P.
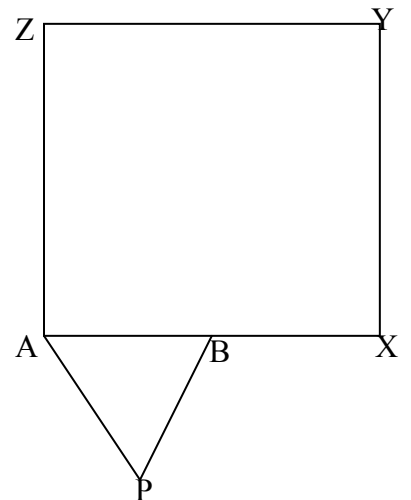
17. 在長方形 ABCD 中，包含了一個圓形的一部分，O 是圓心，半徑爲 10 cm。求陰影部分之面積。(答案以 π 表示）

    The rectangle ABCD contains a part of a circle with center O and radius 10 cm. Find the area of the shaded region.
    (Give the answer in π)
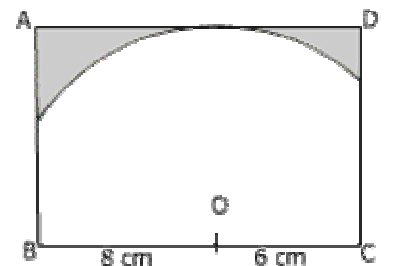
18. 一個邊長 2cm 的立體可分割成 3 個部分，其中兩個部分形狀相同，而第 3 個部分則可由下圖的摺紙圖形所摺成。求所摺成的立體的體積。

    A cube with side 2 cm can be dissected into three parts. Two of the parts are of the same shape. The third part can be folded from the net shown in the right figure. Find the volume of the solid formed from the net.

    2cm
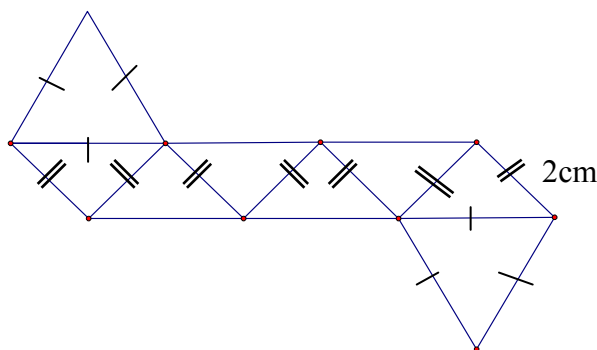
乙部 Part B

把完整的題解和答案寫在答題紙所提供的位置。

Answer the following questions with full solutions on the spaces provided in the answer sheet.

19. 在五個正整數中，每次抽出其中四個相加，可得到 14、15、19 和 20。求該五個數。

There are five positive integers. Taken four at a time, they sum to four distinct values, namely, 14, 15, 19, 20. Find the five positive integers.

20. 下圖中的六邊形包含兩個正方形和兩個三角形。若六邊形及兩個正方形的面積分別為 29、8 和 12，求三角形 ABC 的面積。

The following hexagon consists of 2 squares and 2 triangles. The areas of the hexagon and squares are 29, 8 and 12. Find the area of the triangle ABC.
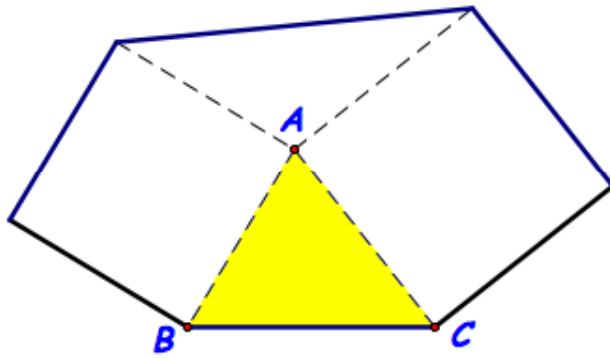
21. (a) 有三人 A、B 和 C 分別被安排坐於三個貼有自己名字的座位上。他們依 A、B、C 的次序入座。第一位坐下的人(A)由於喝醉了，因此他隨意坐下，即他坐於任何一個座位上的機會均等。若座位仍是空的，其餘沒喝醉的兩人(B 和 C)均會按原定的座位坐下；但若座位已被佔用，兩人則會隨意坐下。求第三人(C)坐在屬於自己的座位上的概率。

(b) 若 (a) 部的人數由三人改為十人，其中第一人喝醉了而其餘九人沒喝醉，問第十人坐在屬於自己的座位的概率。

[若某事情共有 N 個機會均等的可能結果，而其中的 m 個結果會有 「E」的情形出現，則 「E」的概率為 $\dfrac{m}{N}$]

【註】

1. 若一件事情有 N 個不相容但機會均等的可能結果，則每個結果的概率為 $\dfrac{1}{N}$。

2. 若一件事件「E」會發生的概率為 x，但「E」的發生有 M 種不相容但機會均等的可能方式，則「E」以每一種方式出現的概率為 $x \cdot \dfrac{1}{M}$。

(a) Three men A, B, C should sit according to their names on the seats. They sit according to the sequence of A and then B and then C. As the first man (A) is drunk, he will sit randomly, that is, it is equally likely that he will sit in any of the three seats. The other two men (B, C) are sober, so each will sit in the correct seat if their seats are not taken, but they will sit randomly if their seats are already taken. What is the chance that the third man (C) will sit down in the seat with his name on?

(b) Same situation as in part (a) but with 10 men instead of 3, the first one who is drunk and the other nine being sober, what is the chance that the tenth man will sit down in the seat with his name on?

[If there are N possible equally likely outcomes and the 'E' happens in m of these outcomes, then the probability of 'E' is $\dfrac{m}{N}$]

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Note:

1. If something can happen in N ways, each being mutually exclusive and equally likely, then each outcome has the probability of $\dfrac{1}{N}$.

2. Given an event 'E' has the probability x of happening and 'E' can happen in M different ways, each being mutually exclusive and equally likely, then the probability of 'E' happening in a particular way is $x \cdot \dfrac{1}{M}$.

乙部完 End of Part B