香港青少年數學精英選拔賽

The Hong Kong Mathematical High Achievers Selection Contest

2004 – 2005

時限：兩小時

Time allowed: 2 hours

除特別指明外，數值答案應用真確值表示。

Unless otherwise specified, numerical answers should be exact.

甲部 Part A

把答案填在答題紙所提供的位置。

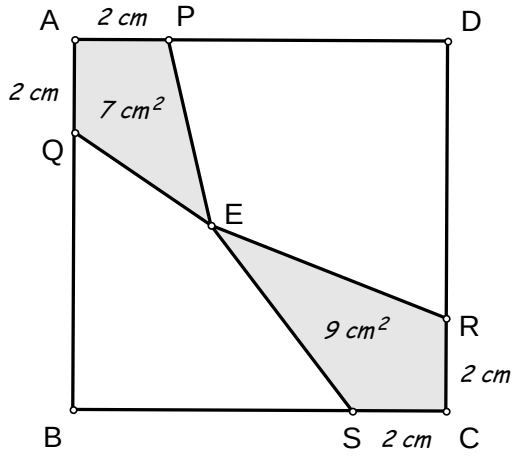
Write the answers on the spaces provided in the answer sheet.

1. Keung and Wai start to run from two diametrically opposite points *A*, *B* on a circular track(i.e. *AB* is a diameter of the circle.), each with different but uniform speeds and in opposite directions. They meet after one minute and each continues to run on his course round the track. How many minutes later will they meet again?

   小強與小偉從圓形跑道對著的兩點 *A*,*B* 起跑(即是說 *AB* 是圓的一條直徑)，各自以不同但均勻的速度迎面而行。一分鐘後兩人迎面相遇，然後各自繼續向前跑，問再過多少分鐘後兩人又再次迎面相遇呢？

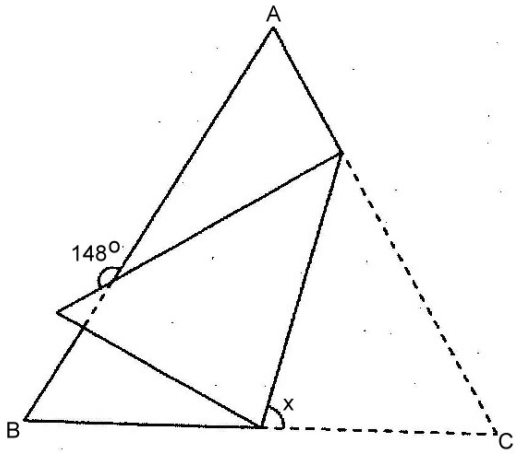
2. In the figure, ABCD is a square. AP = AQ = CR = CS = 2cm. E is a point inside the square such that areas of the quadrilaterals APEQ and CRES are 7 cm^2 and 9 cm^2 respectively. What is the area of the square ABCD?

   圖中，*ABCD* 是一個正方形。AP = AQ = CR = CS = 2cm。E 是正方形內的一點，四邊形 *APEQ* 及 *CRES* 的面積分別是 7 cm^2 及 9 cm^2 。**求正方形** ABCD **的面**積 。



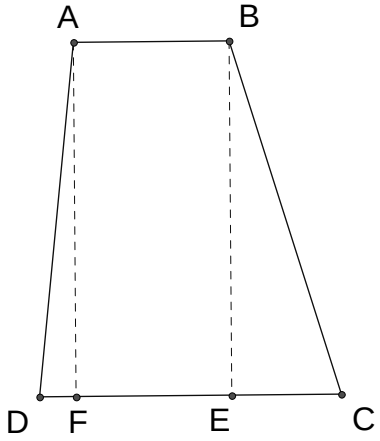
3. In the figure, *ABC* is an equilateral triangle. If it is folded as shown, find the value of x.

圖中三角形 *ABC* 爲一等邊三角形，若把該三角形如圖所示般折疊，求 x 的大小。



4. In the figure, *ABCD* is a trapezium, *AF* and *BE* are perpendicular to *CD*, with $AB + CD = BC$, $BE = 5$ and $DF = 1$. Find the value of $(2AB + 1)(2CD - 1)$.

右圖中，*ABCD* 是一個梯形，*AF* 與 *BE* 垂直於 *CD*，而 $AB + CD = BC$，$BE = 5$ 及 $DF = 1$。求 $(2AB + 1)(2CD - 1)$ 的值。



5. Suppose the perimeter of each triangular face of a tetrahedron is the **same**, and the total length of all **six edges** of the tetrahedron is *L*, what is the perimeter of each triangular face?

若四面體的每個三角面的周界**相同**，而該四面體的**六條側棱**的總長是 *L*, 試求每個三角面的周界。

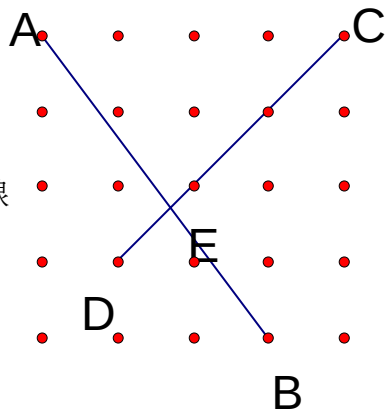
6. It is known that the sum of two positive integers is 45 and their L.C.M. is 168, find these two integers.

已知兩個正整數的和是 45，而它們的最小公倍數是 168，求這兩個數。

7. The sequence $a_1, a_2, a_3, \ldots$ satisfies $a_1 = 20$, $a_9 = 5$, and if $n \geq 3$, a_n is the arithmetic mean of the first $n - 1$ terms. Find a_2.

數列 $a_1, a_2, a_3, \ldots$ 滿足 $a_1 = 20$、$a_9 = 5$ 及當 $n \geq 3$，a_n 是首 $n - 1$ 項的算術平均值。求 a_2。

8. The figure shows 25 lattice points, the shortest distance between the points is 1 unit. Segment *AB* meets segment *CD* at *E*. Find the length of segment *AE*.



圖中有 25 個格點，點與點之間的最短距離爲 1 個單位。線段 *AB* 與線段 *CD* 相交於 *E*。求線段 *AE* 的長度。

9. There is a sequence of nine numbers. The average of the first five numbers is 7, and the average of the last five numbers is 10. If the average of all nine numbers is 9, find the fifth number.

一數列有九個數，前五個數的平均值是 7，後五個數的平均值是 10。如果全部九個數的平均值是 9，求第五個數。

10. Given that $5^4 7^2 = a^b$, where both *a* and *b* are positive integers, find the smallest possible value for $a + b$.

已知$5^4 7^2 = a^b$，其中 *a* 和 *b* 都是正整數。求$a + b$的最小值。

11. A group of workmen takes some bricks from one place to another. If each workman takes y bricks, there will be 12 bricks left. If each of them takes 8 bricks, the last workman just needs to take 7 bricks. What is the number of this group of workmen?

一班工人把一堆磚由一地方搬運到另一地方。若每人搬運 *y* 塊，最後剩下 12 塊。若每人搬運 8 塊，最後一人只需搬 7 塊。問這有多少工人？

12. In ΔABC, BC = 2004, AC= 2005, AB = $\sqrt{\mathbf{2004 + 2005}}$, find sinA×cosC.

在ΔABC 中, BC = 2004, AC= 2005, AB = $\sqrt{\mathbf{2004 + 2005}}$，求 sinA×cosC。

13. Ming went to school when it was 12:00 shown at the clock at home. On his way to school , he looked at the clock in the church and it was 12:20pm. When he reached school, he noted that the time shown at the clock in the school hall was 12:32pm. After staying for 10 minutes at school, he went back home. On his way back, he looked at the church clock again and it was 12:52pm and that the clock at home was 1:16pm when he arrived home. Ming was told that only the time shown at the clock in his home was correct, and if we assume that the speed of Ming remained constant during his journey, find the difference between both times shown at the church clock and the school hall clock and the time shown at the clock at home respectively.

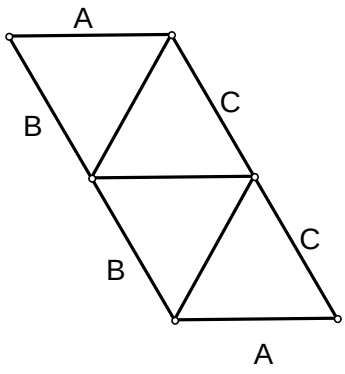
當家裡的大鐘是 12:00 時，小明離家上學。 在上學途中，他看見教會的大鐘是 12:20pm。 到達學校時，他發覺禮堂的大鐘是 12:32pm。他在學校逗留 10 分鐘後，離校返家。在返家途中，他看見教會的大鐘是 12:52pm。回到家裡時，家裡的大鐘是 1:16pm。小明得知只有家裡的大鐘是準確的。假設小明步行速度不變，問教會的大鐘及學校禮堂的大鐘分別與家裡的大鐘相差多少時間。

14. An integer is formed by placing finitely many number 1, 2, 3, … in order and stopping at some stage:
1 2 3 4 5 6 7 8 9 1 0 1 1 1 2 1 3 1 4 1 5 1 6 1 7 1 8 1 9 2 0 2 1 2 2 2 3 … .
The 10^{th} digit is 1. The 30^{th} digit is 2. What is the **2005^{th} digit**?

一整數由 1, 2, 3, … 有限個數順序排列所組成：
1 2 3 4 5 6 7 8 9 1 0 1 1 1 2 1 3 1 4 1 5 1 6 1 7 1 8 1 9 2 0 2 1 2 2 2 3 … 。
該數第十個數字是 1，第三十個數字是 2。問第 2005 個數字是什麼?

15. The following figure shows nets of 2 polyhedrons. Their faces are congruent equilateral triangles. In each net, edges that are joined together are marked with same letters. If the volume of the polyhedron formed from net 1 is 12 cm^3, what is the volume of the polyhedron formed from net 2?

圖中所示的兩個平面圖是由全等的等邊三角形所組成。現對每個平面圖，將有相同英文字母的邊黏合。已知所得的第一個多面體的體積是 12cm^3，求第二個多面體的體積。



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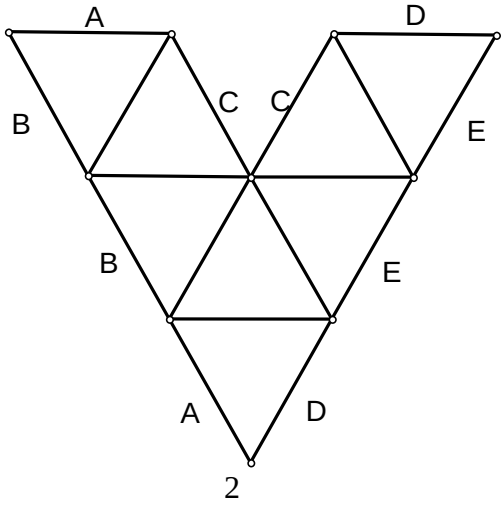


2

甲部完 End of Part A

乙部 Part B

把完整的題解和答案寫在答題紙所提供的位置。

Answer the following questions with full solutions on the spaces provided in the answer sheet.

16. A number A which is divisible by 9 has 2005 digits. The sum of all its digits is a, the sum of all digits of a is b and the sum of all digits of b is c. Find the value of c.

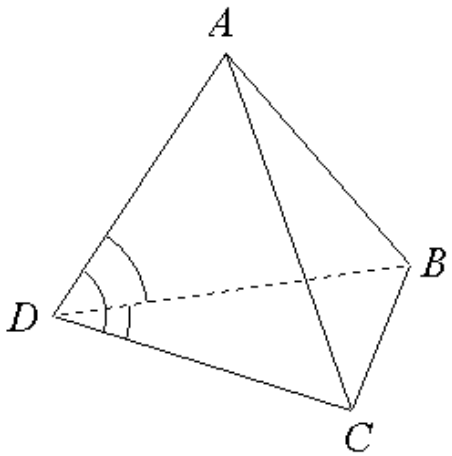
一個 2005 位數 A 能被 9 整除，它的各位數之和爲 a，a 的各位數之和爲 b，b 的各位數之和爲 c，求 c 的值。

17. According to the report of the Hong Kong Observatory, there is a typhoon 220 km due south of Hong Kong. The wind strength at the center is at level 12. The wind movies at a speed of 15 km/h in the direction N30ºE. It is known that for every 20 km away from the center of the typhoon, the wind strength drops by 1 level. If the wind strength measured at Hong Kong reaches level 4 or above, the Hong Kong Observatory will hoist a Typhoon Signal. Now, assuming that the path of the typhoon and the wind strength at its centre remain unchanged, prove that the Hong Kong Observatory must hoist a Typhoon Signal and calculate the length of the period during which the Typhoon Signal is being hoisted.

據天文臺報導，香港的正南方向 220km 處有一颱風，其中心風力爲 12 級，並以 15km/h 的速度向北偏東 30º 的方向移動。已知每距離颱風中心 20km，風力就減弱一級。若香港所受的風力達到四級，天文臺需要懸掛颱風訊號。現假設該颱風的移動方向及中心風力均維持不變，試証明天文臺必會懸掛颱風訊號，並計算颱風訊號需懸掛多久。

18. In a tetrahedron *ABCD*, the three angles (which lie on three different faces) around each **vertex** have a sum equal to **two right angles**. Explain why each edge has the same length as **the edge opposite to it**, i.e. $AB = CD$, $BC = DA$, $CA = BD$.

在四面體 *ABCD* 中，環繞每個**頂點**的三隻角(它們在三個不同的平面上)加起來相等於**兩個直角**。試說明爲什麼每一條棱都與**它所對著的棱**的長度相等，即是 $AB = CD$，$BC = DA$，$CA = BD$。



乙部完 End of Part B