

香港青少年數學精英選拔賽

The Hong Kong Mathematical High Achievers Selection Contest

2003 – 2004

時限：兩小時

Time allowed: 2 hours

除特別指明外，數值答案應用真確值表示。

Unless otherwise specified, numerical answers should be exact.

甲部 Part A

把答案填在答題紙所提供的位置。

Write the answers on the spaces provided in the answer sheet.

1. 化簡 $\sqrt{7 + 2(1 + \sqrt{3})(1 + \sqrt{5})}$ 為 $\sqrt{x} + \sqrt{y} + \sqrt{z}$ 的形式。

Simplify $\sqrt{7 + 2(1 + \sqrt{3})(1 + \sqrt{5})}$ into the form $\sqrt{x} + \sqrt{y} + \sqrt{z}$.

2. 設 $x = \frac{\sqrt{11} - 1}{3}$ ，求 $9x^2 + 6x + 2004$ 的值。

Let $x = \frac{\sqrt{11} - 1}{3}$. Find the value of $9x^2 + 6x + 2004$.

3. 求 $x^2 + y^2 \leq 10$ 的正整數解的數目。

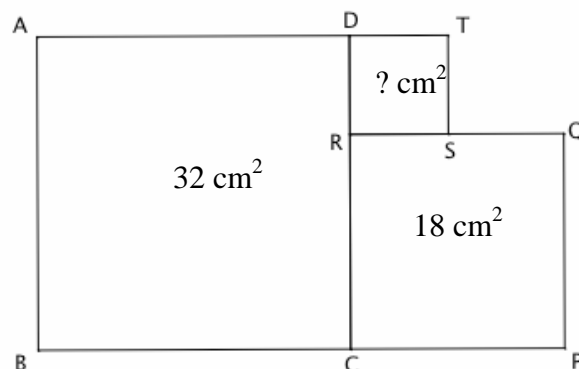
Find the number of positive integral solution of $x^2 + y^2 \leq 10$.

4. 已知 a, b, c ($a < b < c$) 為三個連續奇數且 $b^3 = 4913$ 。求 $a \div c$ 的值。(準確至兩位小數。)

Given that a, b and c ($a < b < c$) are three consecutive odd numbers and $b^3 = 4913$. Find the value of $a \div c$. (Correct to 2 decimal places)

5. 圖中，正方形 $ABCD$ 和 $PQRC$ 的面積分別是 32 cm^2 和 18 cm^2 。求正方形 $RSTD$ 的面積。

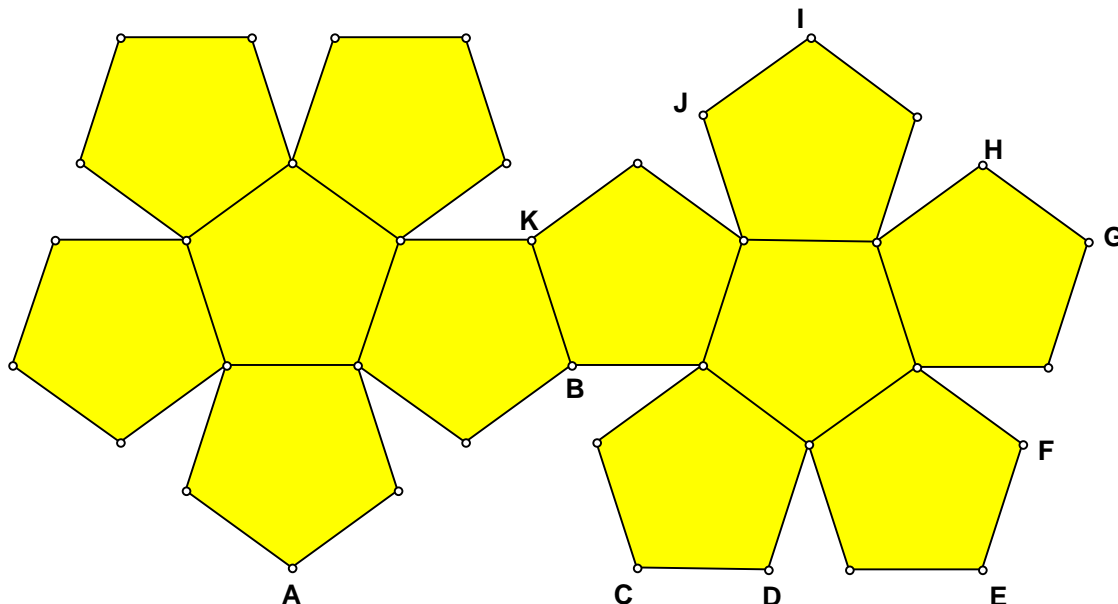
In the figure, areas of the squares $ABCD$ and $PQRC$ are 32 cm^2 and 18 cm^2 respectively. Find the area of the square $RSTD$.



擬題委員會：香港數理教育學會數學組委員 李文生先生（香港大學）卓大偉先生（梁文燕紀念中學（沙田））楊定邦先生（宣道會陳瑞芝紀念中學）趙端銘先生（長沙灣天主教英文中學）
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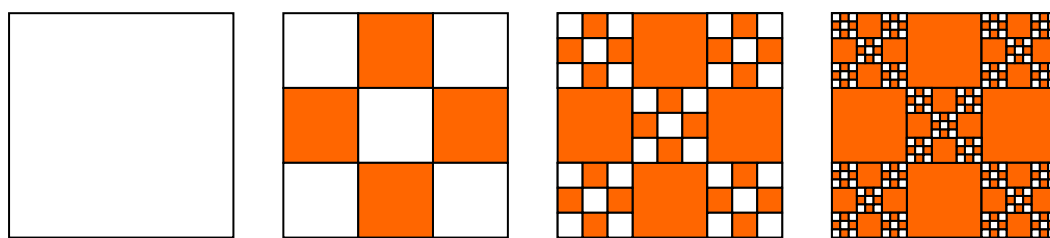
6. 圖示正十二面體的展開圖。這個展開圖被摺成正十二面體後，已標記的頂點 (B 至 K) 中，哪一個會與 A 重疊？

The diagram shows the net of a dodecahedron. Which of the labeled vertices (B to K) will come to A when the net is folded to make the dodecahedron?



7. 根據圖中的規律，第零代的正方形的面積是 1，求第十代的正方形中餘下空白的面積。(答案以指數形式表示。)

The area of the square in stage zero is 1. According to the following pattern, find the remaining (unshaded) area of the square in stage 10. (Answer should be expressed in index form.)



Stage 0
第零代

Stage 1
第一代

Stage 2
第二代

Stage 3
第三代

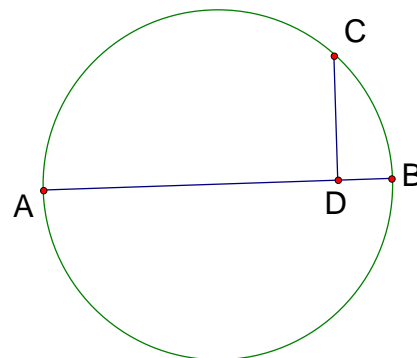
8. 若 $1^3 + 2^3 + 3^3 + \dots + 15^3 = 14400$ ，求 $2^3 + 4^3 + 6^3 + \dots + 30^3$ 。
If $1^3 + 2^3 + 3^3 + \dots + 15^3 = 14400$, find $2^3 + 4^3 + 6^3 + \dots + 30^3$.

9. 一輪船從甲地到乙地要 5 晝夜，而從乙地到甲地要 7 晝夜。設水流方向及速度不變，有一木排從甲地順流漂到乙地，需要多少時間？

A ship travels from A to B in 5 days. If it travels from B to A , it will take 7 days. Suppose the direction and the speed of the current remain constant. How long does it take for a raft floating along the water to go from A to B ?

10. 已知 D 是在圖中直徑 AB 上的一點， $AD = 8$ ， $DB = 4$ ；另 CD 垂直於 AB 。求 CD 的長度。

Given D is a point on the diameter AB , where $AD = 8$, $DB = 4$ and $CD \perp AB$. Find the length of CD .



11. 已知今次區議會選舉的總投票人數比上次多 20%，而投票的男選民人數比上次多 10%，女選民人數則多了 25%。求今次選舉投票的女選民人數佔全部選民人數的比例。

Given that the number of voters who voted in this year's District Council Election increased by 20% from the last time. The number of male voters increased by 10%, and the number of female voters increased by 25%. Find the ratio of number of female voters to number of all voters in this year's Election.

12. 設 $\{a_n\}$ 為一數列，而 $a_1 = 1$ ， $2004a_{n+1} = 1 + 2004a_n$ 。求 a_{2004} 。

Let $\{a_n\}$ be a sequence such that $a_1 = 1$ and $2004a_{n+1} = 1 + 2004a_n$. Find a_{2004} .

13. 某十個整數的平均值、唯一的眾數、中位數和範圍都是等於 10，求該十個整數中最大整數的可能值。

The mean, unique mode, median and range of a collection of ten integers are all equal to 10. Find the largest integer that can be an element of this collection.

14. 設 a_1, a_2, a_3, \dots 為一個由正整數組成的數列，而當 $n \geq 1$ ， $a_1 < a_2 < a_3 < \dots$ 及 $a_{n+2} = a_n + a_{n+1}$ 。設 $a_7 = 200$ ，求 a_8 。

Let a_1, a_2, a_3, \dots be a sequence of positive integers, $a_1 < a_2 < a_3 < \dots$ and $a_{n+2} = a_n + a_{n+1}$ for all $n \geq 1$. If $a_7 = 200$, find a_8 .

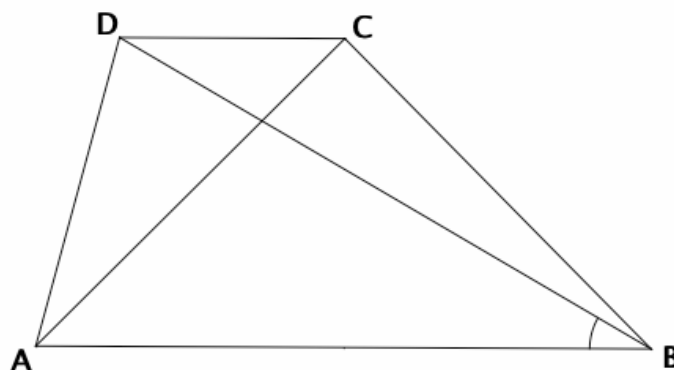
甲部完 End of Part A

乙部 Part B

把完整的題解和答案寫在答題紙所提供的位置。

Answer the following questions with full solutions on the spaces provided in the answer sheet.

15. 圖示一個梯形， DC 與 AB 平行， $AC \perp CB$ 、 $AC = CB$ 及 $BA = BD$ 。求 $\angle ABD$ 。
In the following trapezium, DC is parallel to AB ; $AC \perp CB$; $AC = CB$ and $BA = BD$. Find $\angle ABD$.

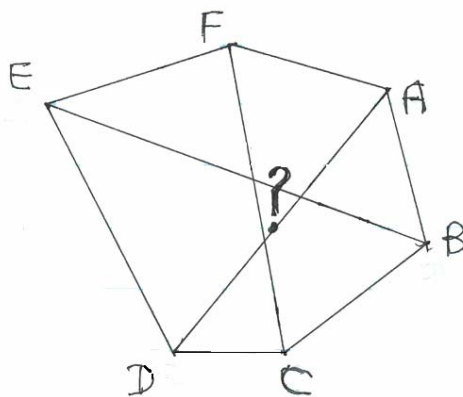


16. $ABCDEF$ 是個凸六邊形，每條對角線 AD , BE , CF 都把六邊形的面積平分（見圖）。證明那三條對角線交於一點。

[提示：三角形邊長是 a 、 b ，夾角是 θ ，則面積等於 $\frac{1}{2}ab \sin \theta$ 。]

$ABCDEF$ is a convex hexagon. Each of the three diagonals AD , BE , CF bisects the area of the hexagon (see figure). Show that the three diagonals intersect at a common point.

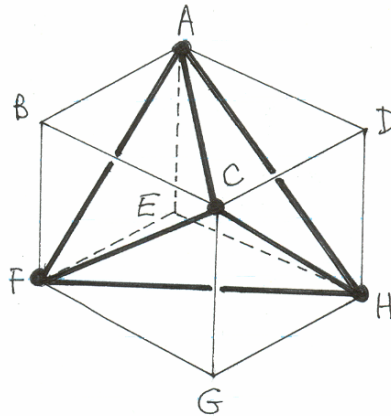
[Hint: The area of a triangle of sides of length a , b and included angle θ is equal to $\frac{1}{2}ab \sin \theta$.]



17. 正立方體 $ABCDHEFG$ 內含正四面體 $ACHF$ (見圖)。計算
 $\frac{ACHF \text{ 的體積}}{ABCDHEFG \text{ 的體積}}$ 。

The cube $ABCDHEFG$ contains the regular tetrahedron $ACHF$ (see figure).

Calculate $\frac{\text{Volume of } ACHF}{\text{Volume of } ABCDHEFG}$.



18. 找三個相連的偶數，它們的乘積等於 $37*****2$ 。

Find three consecutive even integers with their product equal to $37*****2$.

乙部完 End of Part B