

香港青少年數學精英選拔賽

The Hong Kong Mathematical High Achievers Selection Contest

2002-2003

限時：兩小時

Time allowed: 2 hours

如有需要，可用 $\pi$ 表示答案中的數值

Express the answers in terms of  $\pi$ , if necessary.

答案填在答題紙所提供的位置

Write the answers on spaces provided on the answer sheet.

1. 淦頌每天用紙 3 張，思恒每天用紙 5 張，每人每次購買一疊紙共 20 張。他們每逢發現翌日不夠紙用，便會去購買紙張。第一天淦頌和思恒一同去購買紙張，下一次兩人一同去購買紙張是那一天？

Kam-chung uses 3 sheets of paper per day and Sze-hang uses 5 sheets of paper per day. They go to purchase a batch of 20 sheets of paper on the day they discover that there are not enough sheets for the next day. If Kam-chung and Sze-hang go to purchase together on the first day, on which day will they next to go to purchase together?

2. 有五個數  $a$ 、 $b$ 、 $c$ 、 $d$ 、 $e$ ，已知  $a+b=3$ 、 $b+c=6$ 、 $c+d=9$ 、 $d+e=7$  及  $e+a=5$ ，求  $a$ 、 $b$ 、 $c$ 、 $d$ 、 $e$ 。

Five numbers  $a, b, c, d, e$  satisfy  $a+b=3, b+c=6, c+d=9, d+e=7$  and  $e+a=5$ . Find  $a, b, c, d, e$ .

3. 求  $\frac{2003125}{2003125^2 - (2003124)(2003126)}$  的值。

Evaluate  $\frac{2003125}{2003125^2 - (2003124)(2003126)}$ .

4. 求 40 位數字 20032 00320 03200 32003 20032 00320 03200 32003 除以 9 的餘數。

Find the remainder when the 40-digit number 20032 00320 03200 32003 20032 00320 03200 32003 is divided by 9.

5. 求  $18^{42}$  的個位數字。

Find the unit digit of the number  $18^{42}$ .

6. # 這符號被定義為

$$1 \# 2 = 44、$$

$$3 \# 4 = 36、$$

$$7 \# 5 = 26、$$

$$12 \# 10 = 6, \text{ 及}$$

$$4 \# 9 = 24;$$

求  $6 \# 8$  的值。

The symbol '#' is defined as

$$1 \# 2 = 44,$$

$$3 \# 4 = 36$$

$$7 \# 5 = 26,$$

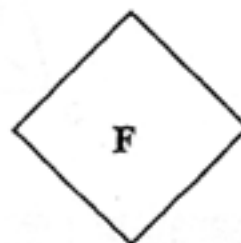
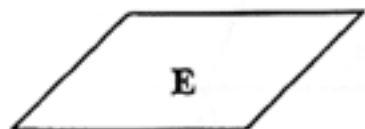
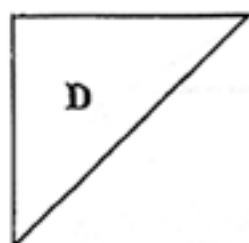
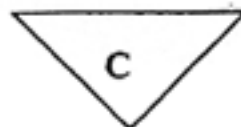
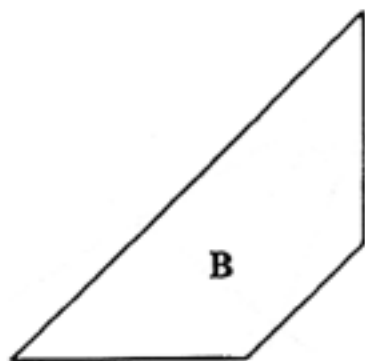
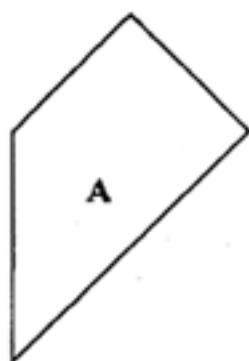
$$12 \# 10 = 6, \text{ and}$$

$$4 \# 9 = 24.$$

Find the value of  $6 \# 8$ .

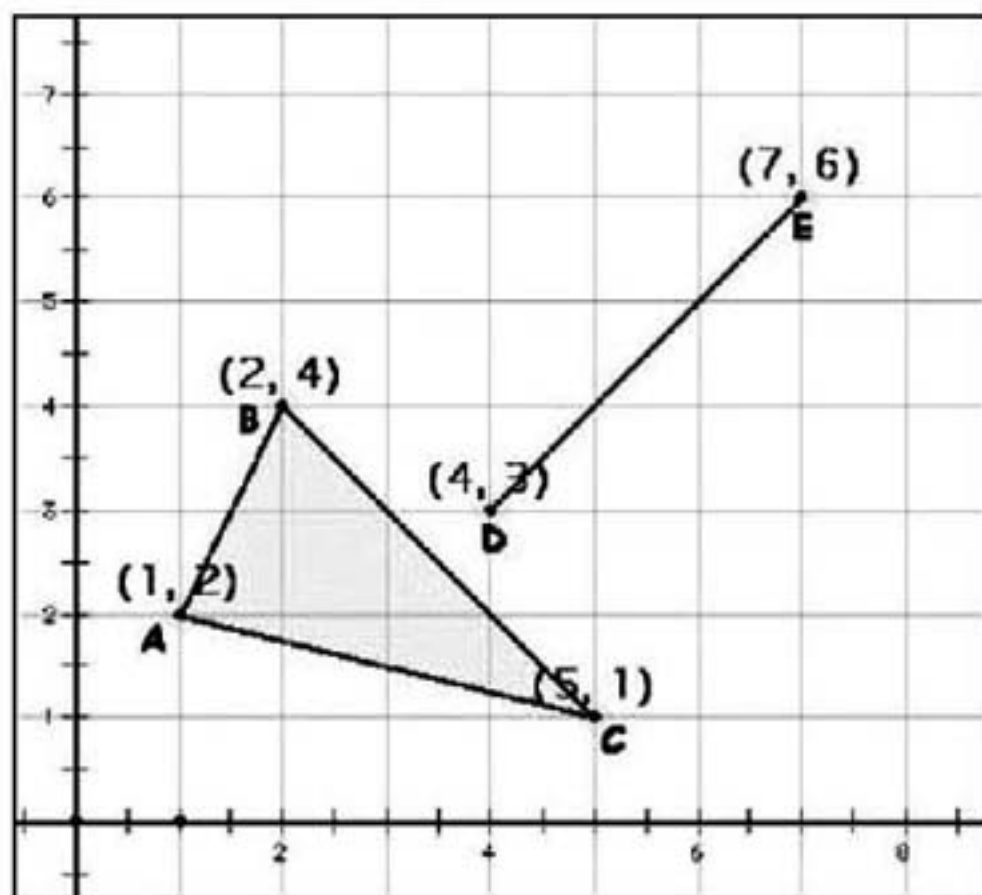
7. 下列那些形狀可以拼合成右方的陰影面積？寫下所有可能的組合，每個組合中每個形狀只可使用一次。

Which of the shapes can be put together to make the shaded area? Write down all possible combinations. In each combination each shape can only be used once.



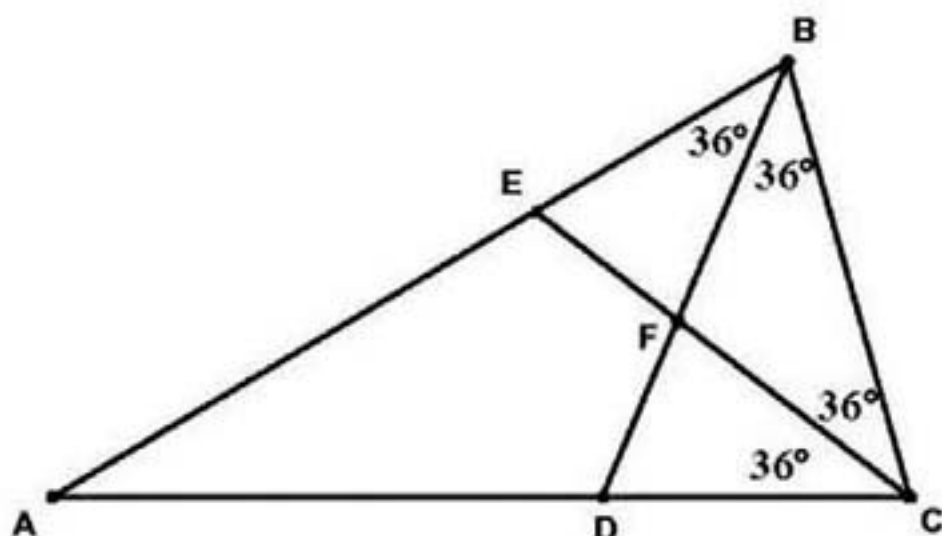
8. 如果在下圖中加  $F$  點使  $\triangle ABC$  及  $\triangle DEF$  成全等三角形，求  $F$  所有的可能座標。

$F$  is a point in the following graph such that  $ABC$  and  $DEF$  are two congruent triangles. Find all possible coordinates of  $F$ .



9. 下圖中共可找到多少個等腰三角形？

How many isosceles triangles can be found in the following figure?

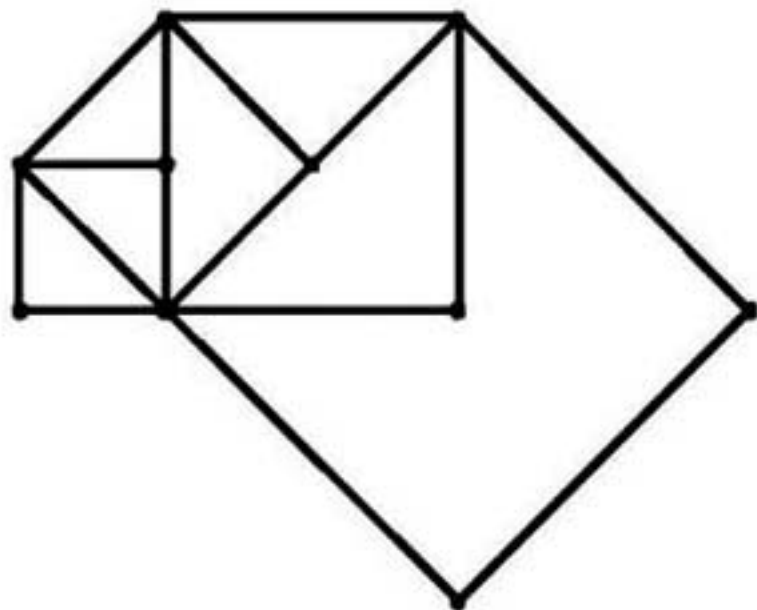


10. 某實心正方體的邊長是 1 單位，所有離該立方體表面 1 單位的點圍著一個實心的形狀，求該實心形狀的體積，以  $\pi$  表示答案。

A solid cube has edges of length 1 unit. The collection of points precisely 1 unit away from the surface of the cube encloses a solid shape. Find the volume of the solid shape in terms of  $\pi$ .

11. 圖示一組互相重疊的正方形；如果最小的正方形的面積是 3.5 平方單位，求最大的正方形的面積。

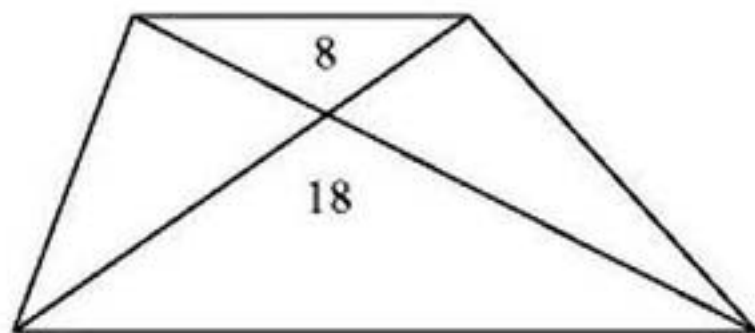
The following figure shows a set of overlapping squares. If the area of the smallest square is 3.5 square units, find the area of the largest square.



12. 求 1 至 100000000 間所有整數的數字的總和。(例如  $123 \rightarrow 1 + 2 + 3$ )  
Find the sum of all the digits in all the integers from 1 to 100000000.(e.g.  $123 \rightarrow 1 + 2 + 3$ )

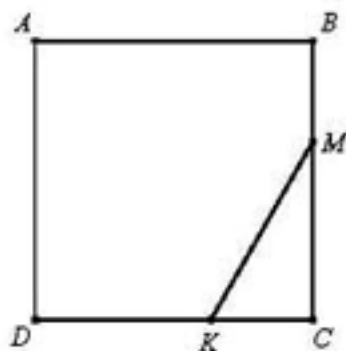
13. 圖中的梯形被它的對角線分成四個三角形，而上下兩個三角形的面積分別是 8 及 18 平方單位，求該梯形的面積。

A trapezium is divided into four triangles by its diagonals as shown in the diagram. The top and bottom triangles have areas 8 and 18 square units respectively. Find the area of the trapezium.

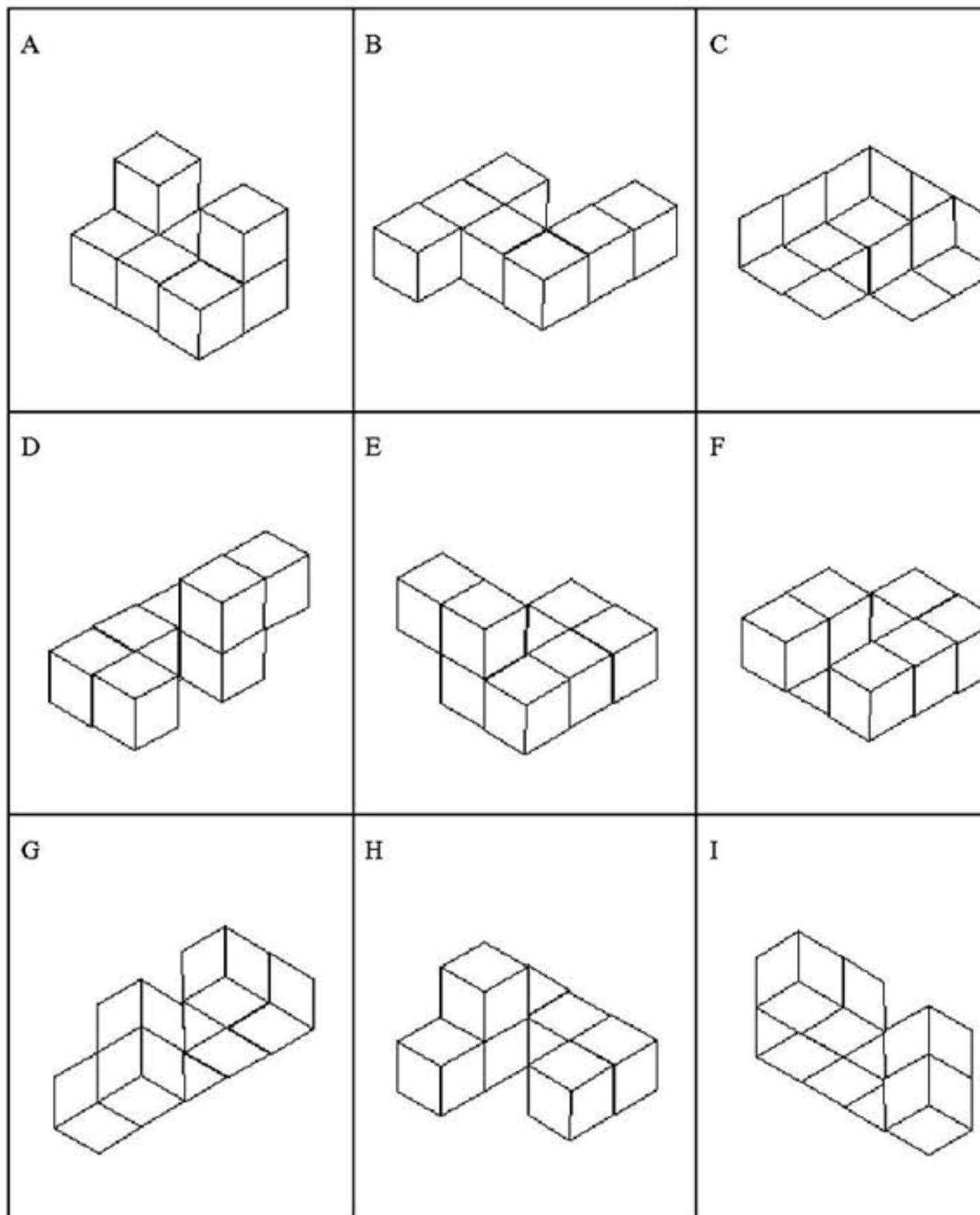


14. 圖中， $M$  與  $K$  分別是正方形  $ABCD$  上  $CB$  及  $CD$  上的點，如果正方形周界是  $\triangle CMK$  的周界的兩倍，求  $\angle MAK$ 。

In the figure,  $M$  and  $K$  are points on  $CB$  and  $CD$  respectively of the square  $ABCD$ . If the perimeter of the square equals two times the perimeter of  $\triangle CMK$ , find  $\angle MAK$ .

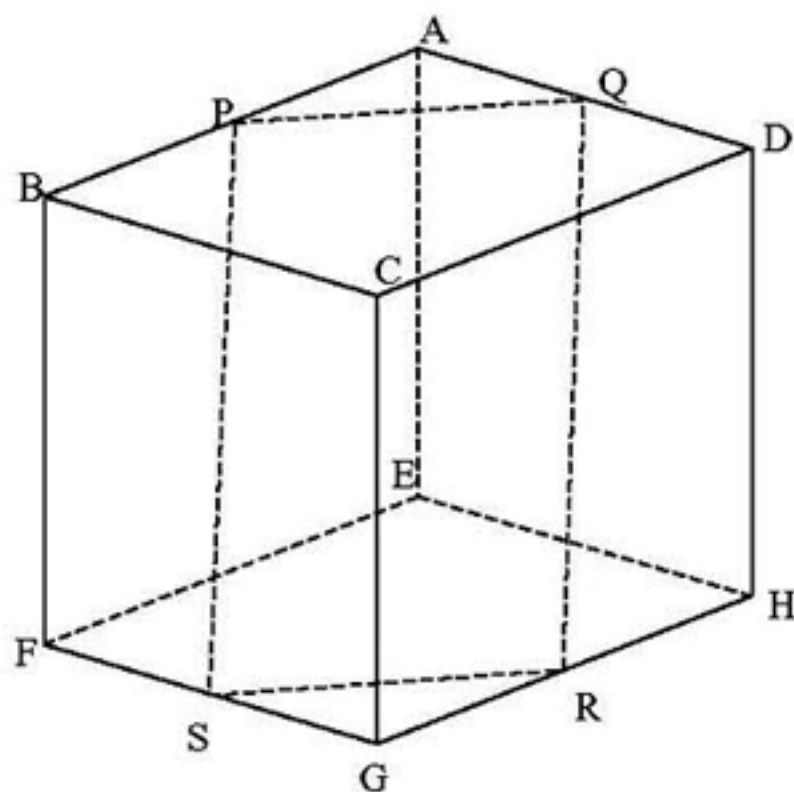


15. 下列其中四幅圖是某同一立體由不同角度被觀察的圖樣，找出該四幅圖樣。  
 Four of the following figures are different views of the same solid. Find these four figures.



16. 圖示一正立方體， $P$ 、 $Q$ 、 $R$ 、 $S$  分別是  $AB$ 、 $AD$ 、 $GH$ 、 $GF$  的中點，求  $PR$  與  $SQ$  間的銳角。

The figure shows a cube.  $P$ ,  $Q$ ,  $R$  and  $S$  are midpoints of the edges  $AB$ ,  $AD$ ,  $GH$  and  $GF$  respectively. Find the acute angle between  $PR$  and  $SQ$ .



17. 某晚上有四個傷兵需要橫過一條破橋以逃離敵方炮火，該破橋每次最多只能承托兩個士兵，當兩個士兵一起過橋時，他們必須以較慢的士兵的速度行走；該四個士兵只有一支照明燈；他們每次都必須攜帶照明燈以安全地過橋；如果該四個士兵各需 1、2、4，及 6 分鐘時間過橋，求他們全部安全過橋的最短時間。

Four wounded soldiers have to cross a damaged bridge at night to escape from enemy fire. The bridge can only take two soldiers at a time; when two soldiers cross together, they have to travel at the speed of the slower member. The four soldiers have only one torch between them: to cross safely the torch has to go across each time. If the four soldiers individually take 1, 2, 4 and 6 minutes to cross the bridge, find the minimum time needed for all four to cross safely.

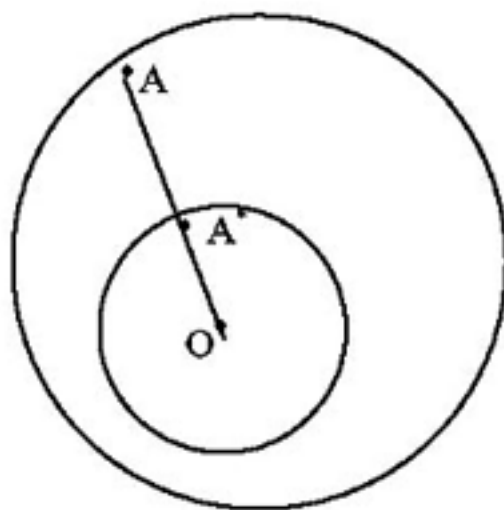
乙部 Part B

把完整的題解和答案寫在答題紙所提供的位置。

Answer the following questions completely on the spaces provided in the answer sheet.

18.  $C_1$  是一個以  $O$  為圓心、 $r$  為半徑的圓。已知  $C_1$  落在另一個半徑為  $R$  的圓  $C_2$  中。若  $A$  是  $C_2$  內及  $C_1$  外的一點，而  $A^*$  是  $OA$  上的一點使  $OA \times OA^* = r^2$ 。解釋為何  $AA^*$  不大於  $4(R-r)$ 。

A circle  $C_1$  with centre  $O$  and radius  $r$  lies inside another circle  $C_2$  with radius  $R$ .  $A$  is a point lying inside  $C_2$  and outside  $C_1$ .  $A^*$  is a point on  $OA$  satisfying  $OA \times OA^* = r^2$ . Explain why  $AA^*$  cannot exceed  $4(R-r)$ .

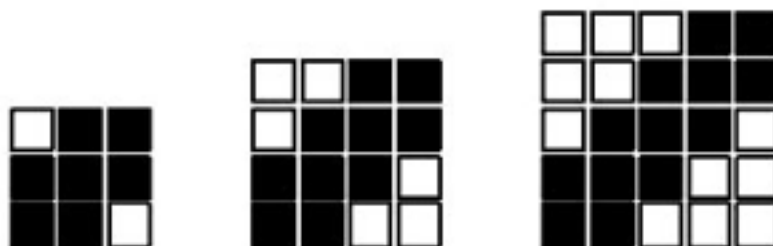


19. 144 是一個有趣的數，它的三個數字的和乘以它的三個數字的積恰巧是它自己，試在 100 和 200 內尋找另一個滿足這性質的數。你必須解釋尋找的過程。  
The number 144 enjoys an interesting property, namely, the sum of its three digits multiplied by the product of its three digits yields the number itself. Find another number between 100 and 200 which enjoys the same property. You must explain your workings.



20. 下列三個正方形階磚圖案都是由黑色及白色的階磚組成，某較大的正方形階磚圖案亦以相似的方法組成，而其中共有 58 塊黑色階磚，求該正方形階磚圖案中白色階磚的數目。你必須寫下詳細的解。

The following three square tile patterns consist of black and white tiles. A larger square pattern is built in similar way and consists of 58 black tiles. Find the number of white tiles in that square pattern. You should provide a detailed solution.



乙部完 End of Part B