香港青少年數學精英選拔賽

The Hong Kong Mathematical High Achievers Selection Contest

2001-2002

時限：兩小時

Time allowed: 2 hours

除特別指明外，數值答案應用真確值表示。

Unless otherwise specified, numerical answers should be exact.

甲部 Part A

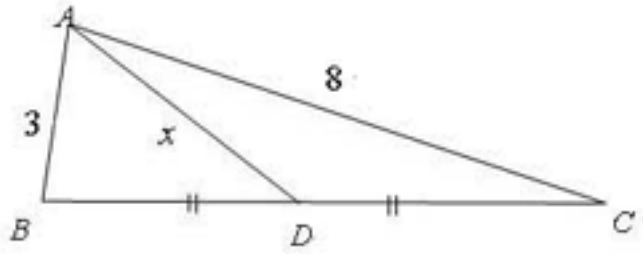
把答案填在答題紙所提供的位置。

Write the answers on the spaces provided in the answer sheet.

1. The volume of a cube is 36 cm^3. A regular octahedron is formed by joining the centers of adjoining faces of the cube. What is the volume of the octahedron?
一正立方體的體積是 36 cm^3，把相鄰的面的中點聯起，構作一個正八面體。求這個正八面體的體積。

2. Let $ABCD$ be a parallelogram of area 15 cm^2 with $AB = 4$ cm and $BC = 5$ cm. E, F, G and H lie on AB, BC, CD and DA respectively, with $AE = BF = DG = AH = 3$ cm. Find the area of the quadrilateral $EFGH$.
設 $ABCD$ 爲一個面積 15 cm^2 的平行四邊形，$AB = 4$ cm，$BC = 5$ cm；E、F、G 及 H 分別是在 AB、BC、CD，及 DA 上的點，而 $AE = BF = DG = AH = 3$ cm。求四邊形 $EFGH$ 的面積。

3. The figure shows a triangle ABC with $AB = 3$ and $AC = 8$. If D is the mid-point of BC and AD = x, where x is an integer, find all the possible values of x.
圖示三角形 ABC，$AB = 3$，$AC = 8$。D 是 BC 之中點，$AD = x$，而 x 爲一整數。求 x 所有可能的值。



4. 1□△ is a three-digit integer that is equal to the sum of cubes of its digits, that is
$$1\square\triangle = 1^3 + \square^3 + \triangle^3 .$$
Find this integer.
1□△ 是一個三位數，而其值與其位值的三次冪總和相同，即
$$1\square\triangle = 1^3 + \square^3 + \triangle^3 .$$
求這三位數。

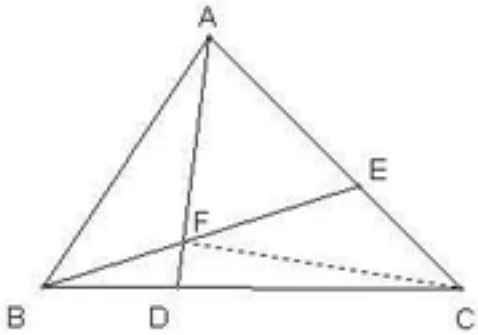
5. A rectangle has an area of 960 cm^2 and sides in centimeters which are multiples of 4. How many rectangular shapes satisfy this condition? (note : rectangles of dimensions 4cm ×240cm and 240cm × 4cm are counted as one shape.)
一長方形之面積爲 960 cm^2，而其邊長是 4cm 之倍數。有多少個不同形狀的長方形滿足以上的條件？（注意：長度爲 4cm ×240cm 及 240cm × 4cm 的長方形作同一種形狀計。）

6. Let a, b, c and d be integers. One of them is an even number, and the others are odd numbers. The possible sums of adding any two numbers are 66, 75, 87, 98, 110, 119. Find the sum of the three odd numbers.
a、b、c 及 d 是四個整數，其中一個是偶數，其餘是奇數。從中任意選兩個相加，得出的和是 66、75、87、98、110、119。求三個奇數的和。

7. Find the unit digit of 7^{2002}.
求 7^{2002} 的個位數。

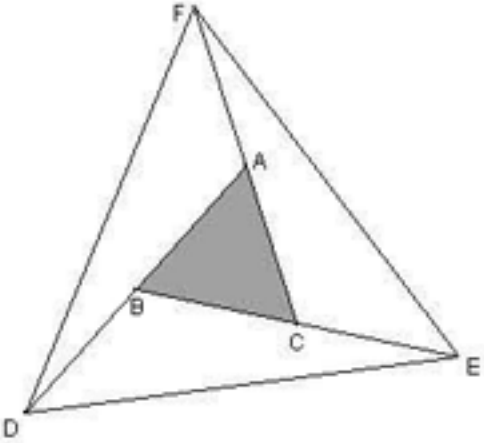
8. x is a three-digit number. Its digits are all different and not equal to zero. Rearrange the digits of x to get five other numbers. These five numbers and x are added together to get 4218. Find the sum of digits of x.
x 是一個三位數，它的各個位值均不同且不等於零。重新排列 x 的每一位數字可得另外五個數。把這五個數及 x 相加可得 4218。求 x 的位數的和。

9. In triangle ABC, $BD : BC = CE : CA = 1 : 3$. Let F be the intersection of AD and BE. Find $AF : FD$.
三角形 ABC 中 $BD : BC = CE : CA = 1 : 3$，設 F 爲 AD 及 BE 的交點，求 $AF : FD$。

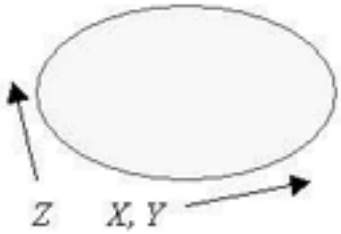


10. It is known that the cost of tickets for 7 children and 2 adults to go to Disney World Hong Kong is \$4100. On the other hand, the cost of tickets for 20 children and 3 adults is \$9150. However, a 10% discount is given when purchasing more than 10 tickets for children (discount for children's tickets only.). Find the cost of ticket for an adult.
七張香港迪士尼小童票及兩張成人票共需\$4100，而二十張小童票及三張成人票則共需\$9150；但是，購買超過十張小童票就可得九折優惠 (優惠只給予小童票)。求一張成人票的價錢。

11. ABC is a triangle. CAF, ABD, BCE are straight lines such that $AF = AC$, $BD = BA$, $EC = CB$. If the area of triangle ABC is 1 square unit. Find the area of triangle DEF.
ABC 是一三角形，CAF、ABD、BCE 都是直線而 $AF = AC$，$BD = BA$ ， $EC = CB$。如果三角形 ABC 的面積是 1 平方單位，求三角形 DEF 的面積。



12. A road is built around the perimeter of a reservoir. Three trailwalkers X, Y, Z began their walk around the reservoir at the same place. X and Y went in counter-clockwise direction, and Z went in clockwise direction. The speed of X and Y are 160m/min and 130m/min respectively. Z met X after 20 minutes. He then met Y after 2 minutes. Find the perimeter of the reservoir.



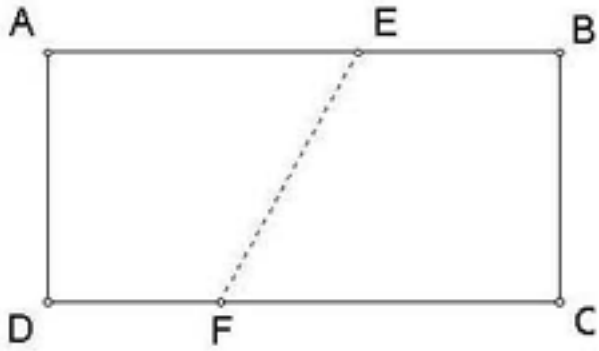
沿著某水塘的周界有一條小路，三個行山人士 X、Y，和 Z 在同一地點出發，X 和 Y 沿反時鐘方向走，而 Z 則沿順時鐘方向走，X 和 Y 的步速分別是每分鐘 160m 和每分鐘 130m，Z 在二十分鐘後遇見 X，兩分鐘後再遇見 Y，求水塘的周界長。

13. If $x+\frac{1}{x}=5$, find $\frac{x^2}{x^4+x^2+1}$.

若 $x+\frac{1}{x}=5$，求 $\frac{x^2}{x^4+x^2+1}$。

14. A rectangular piece of paper is shown in the figure, $AD=9$ cm, $AB=12$ cm. The piece of paper is folded along EF such that A and C touch each other. Find the length of EF.

圖示一張長方形紙，$AD=9$ cm, $AB=12$ cm。把這張紙沿 EF 對摺令 A、C 重合，求 EF 的長度。

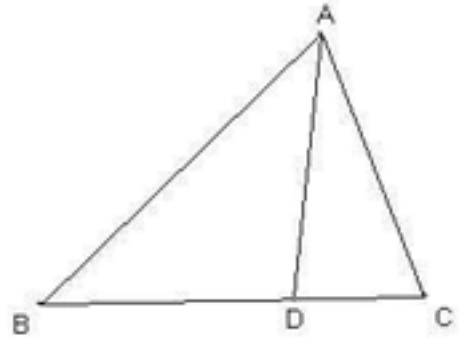


15. 1111122222 can be factorized into the product of two consecutive positive integers. Find the larger factor.

1111122222 可分解成兩個連續正整數的積，求較大的因數。

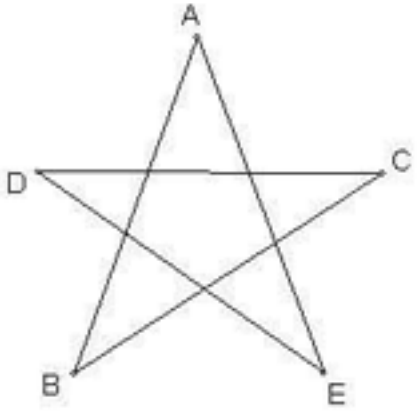
16. In triangle ABC, D is a point on BC such that $BD=DA=AC$, and $\angle BAC=63°$. Find $\angle DAC$.

在三角形 ABC 內，BC 上有一點 D 令 $BD=DA=AC$，而 $\angle BAC=63°$。求 $\angle DAC$。



17. In a star-shaped figure $ABCDE$, it is known that $\angle B=\angle C=\angle D=\angle E=34°$. Find $\angle A$.

圖示爲一五角星，若 $\angle B=\angle C=\angle D=\angle E=34°$，求 $\angle A$。



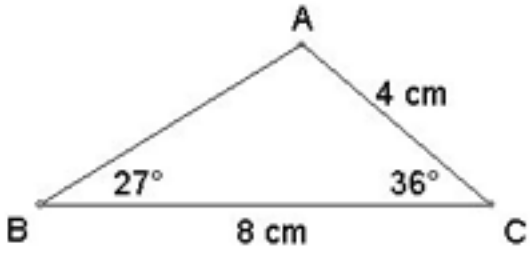
甲部完 End of Part A

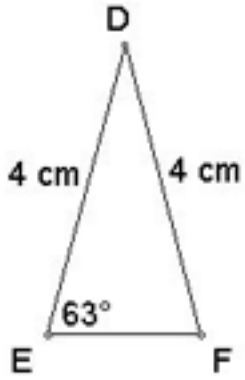
乙部 Part B

把完整的題解和答案寫在答題紙所提供的位置。

Answer the following questions completely on the spaces provided in the answer sheet.

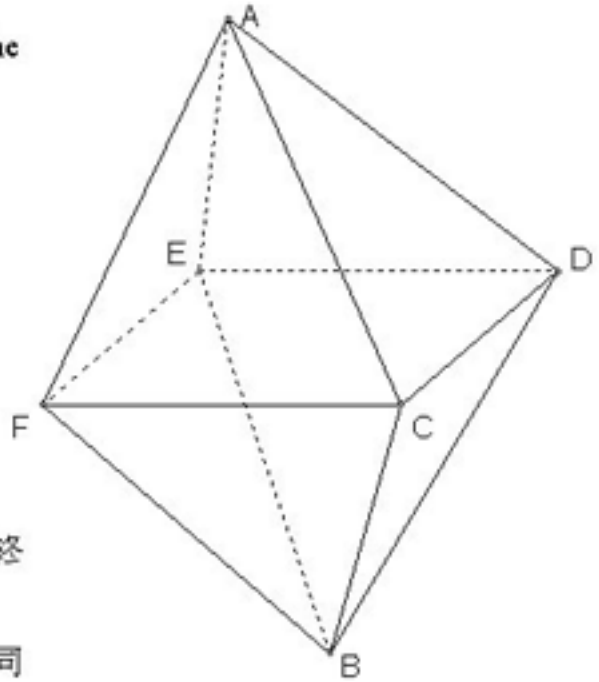
18. Triangle ABC and DEF are as shown in the figure. The area of triangle DEF is 6.4 cm^2. Find the area of triangle ABC.
圖示三角形 ABC 及 DEF。三角形 DEF 的面積是 6.4 cm^2。求三角形 ABC 的面積。





19. There are 11 equilateral triangles. One of them has side 5 cm. Three of them have side 4 cm. Four of them have side 3 cm. Three of them have side 2 cm.
(a) Arrange these equilateral triangles into a large equilateral triangle without spaces, folding, or overlapping.
(b) Find also the length of side of this large equilateral triangle.
現有 11 個正三角形，其中一個邊長 5 cm，三個邊長 4 cm，四個邊長 3cm，三個邊長 2 cm。
(a) 把這些三角形拼成一個沒有縫隙的大正三角形，各三角形不可重疊及不可折疊。
(b) 另求這個大正三角形的邊長。

20. An octahedron $ACDEFB$ is shown in the figure. Let A be the starting point and endpoint of paths along the edges of the octahedron. The paths cannot go through the same vertex twice(except the starting point and endpoint A). For example, $A \to C \to F \to C \to D \to A$ is not allowed. However, $A \to C \to D \to A$ and $A \to D \to C \to A$ are considered two different paths.
(a) Find the number of such paths *not passing through B*.
(b) Find the number of such paths *passing through B*.
圖示正八面體 $ACDEFB$，構作路線由 A 作起點沿邊前進，最後回到 A，這些路線不能重覆通過同一點(起點及終點 A 除外)。例如 $A \to C \to F \to C \to D \to A$ 是不容許的；但 $A \to C \to D \to A$ 與 $A \to D \to C \to A$ 則是兩條不同的路線。
(a) 求不通過 B 的路線總數。
(b) 求通過 B 的路線總數。



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