香港青少年數學精英選拔賽

The Hong Kong Mathematical High Achievers Selection Contest

1999-2000

限時: 兩小時
Time allowed: 2 hours
Express the answers in terms of π, if necessary. 如有須要，可以 π 表示答案中數值。

Part A (甲部)
請將答案填在答案紙的指定位置上。
Write the answers on the spaces provided in the answer sheet.

1. Evaluate (654321)(654321) – (654323)(654319).

   計算 (654321)(654321) – (654323)(654319)。

2. If $\frac{a+b}{a-b}=\frac{7}{4}$, find the value of $\frac{a^2}{b^2}$.

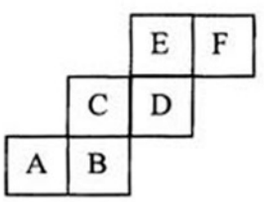
   若 $\frac{a+b}{a-b}=\frac{7}{4}$ ，求 $\frac{a^2}{b^2}$ 之值。

3. If a, b and c are digits from 0 to 9 for which $\mathbf{9b2}-\mathbf{48c}=\mathbf{a73}$, find the value of a + b + c.

   若 a, b 及 c 爲 0 至 9 之數字而使等式 $\mathbf{9b2}-\mathbf{48c}=\mathbf{a73}$ 成立，求 a + b + c 之值。

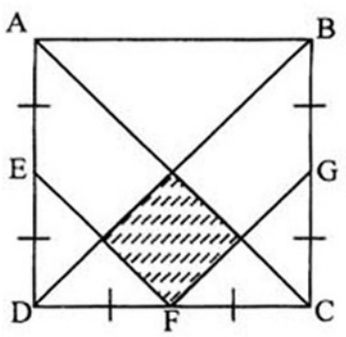
4. When the figure is folded into a cube. Find the letter that is on the face opposite to A.

   把右邊的圖形摺成立方体後, 那一個字母會在 " A " 的對面？



5. The figure shows a square ABCD. E, F and G are mid-points of AD, DC and CB respectively. Find the ratio of the area of the shaded region to the area of ABCD.

   ABCD 是一個正方形，E、F 和 G 分別是 AD，DC 和 CB 的中點。
   求陰影部份面積與 ABCD 面積的比。



6. If the ratio of the difference, sum and product of two numbers is 1 : 7 : 24, find the value of the product of these two numbers.

   兩數之差，和與積之比爲 1 : 7 : 24 ， 求此兩數之積。

7. In a class of no more than 40 students there are exactly 10% more girls than boys. How many girls are there?

一班少於 40 人的學生中，女生人數剛好比男生人數多百份之十。求女生人數。

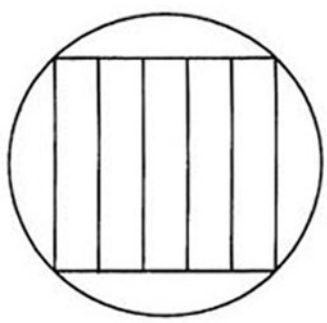
8. Given $a = 999 \times 998998999$ and $b = 998 \times 999999998$. Find the value of $a - b$.

已知 $a = 999 \times 998998999$ 及 $b = 998 \times 999999998$。求 $a - b$ 之值。

9. Find the value of $1-2-3+4+5-6-7+8+9-\ldots\ldots+1996+1997-1998-1999$.

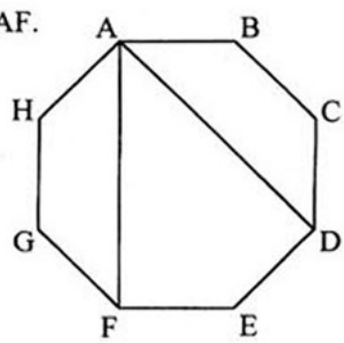
求 $1 - 2 - 3 + 4 + 5 - 6 - 7 + 8 + 9 - \ldots\ldots + 1996 + 1997 - 1998 - 1999$ 之值。

10. The figure shows a square circumscribed by a circle. If this square is divided into five equal rectangles and each rectangle has perimeter 24cm, find the area of the circle. [Give the answer in terms of π]



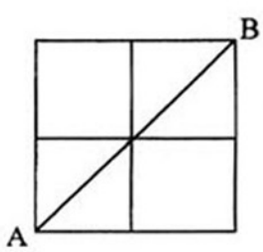
如圖，一個圓內接正方形被分成五個全等長方形。已知每一個長方形的周界爲 24cm ，問這個正方形的外接圓的面積是多少？
[答案須以π表示]

11. The figure shows a regular octagon ABCDEFGH. Find $\angle DAF$.



如圖，ABCDEFGH 是一個正八邊形，求$\angle DAF$。

12. In how many ways can you get from A to B if you are only allowed to move from left to right or vertically/diagonally upwards.

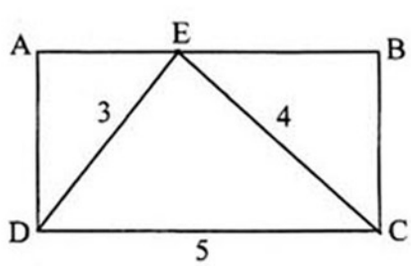


若只能夠從左至右，從下至上或沿對角線向上移動，有多少不同的路徑可以由 A 走到 B？

13. Find a four-digit number $\overline{abcd}$, which is equal to $\overline{dcba}$ when multiplied by 9.

一四位數 $\overline{abcd}$ 乘 9 後等於 $\overline{dcba}$ 。求這四位數。

14. In the figure, ABCD is a rectangle. DC = 5. E is a point on AB such that DE = 3 and EC = 4. Find the perimeter of rectangle ABCD.



圖中 ABCD 爲一矩形，DC = 5，E 爲 AB 上一點，而 DE = 3、 EC = 4。求矩形 ABCD 之周界。

End of Part A 甲部完

Part B (乙部)

以下題目，請於答案紙上指定位置完整作答。

Answer the following questions completely on the spaces provided in the answer sheet.

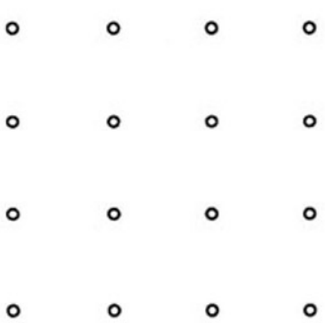
15. Which value is bigger : 2^{1000} or 3^{750} ? Explain.

    2^{1000} 與 3^{750} 中，那一個數值較大？請解釋。

16. By using the dots in the given square grid as vertices, how many squares of different areas can be drawn?
    Draw each of these squares on the square grids provided on your answer sheet.

    How many right angled triangles, which are not isosceles, can be drawn on the square grid, using the dots as vertices? Draw each of these triangles on the square grids provided on your answer sheet.

    For figures with same area, you need to draw only one of them.



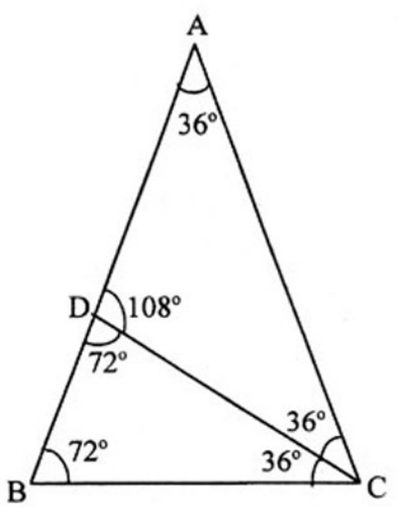
    以上圖中方格內的點作爲頂角，可以畫出多少個不同面積的正方形?
    試將這些正方形逐一畫在答題紙上所提供的方格內。

    另外，以這些點作爲頂角，又能作出多少個不同面積的非等腰直角三角形？
    試將這些三角形逐一畫在答題紙上所提供的方格內。

    同面積的圖形，只須畫一個。

17. In the figure an isosceles triangle ABC can be divided into 2 smaller isosceles triangles. Suggest all other isosceles triangles, which can also be divided into 2 isosceles triangles. Draw these triangles in your answer sheet and indicate the sizes of all angles in each triangle.

    圖中等腰三角形 ABC 能被分成兩個小等腰三角形。試求出其餘具以上條件的等腰三角形，即本身亦能被分爲兩個小等腰三角形。將每個三角形畫在答題紙上，並指出每個角的大小。



End of Part B 乙部完