

香港青少年數學精英選拔賽

The Hong Kong Mathematical High Achievers Selection Contest

1998-1999

限時：兩小時

Time allowed: 2 hours

這份卷共分爲甲部及乙部兩個部份。所有問題必須作答。

This paper is divided into two parts in which all the questions should be answered.

如有須要，答案可以以 π 表示。

If necessary, you may express the answers in terms of π .

Part A (甲部)

Write the answers on the spaces provided in the answer sheet.

請將答案填在答案紙上指定的位置上。

1. Calculate $\sqrt[3]{(8.5^2 - 3.5^2)(8.5^2 - 7 \times 8.5 + 3.5^2)(8.5^2 + 17 \times 3.5 + 3.5^2)}$.

求 $\sqrt[3]{(8.5^2 - 3.5^2)(8.5^2 - 7 \times 8.5 + 3.5^2)(8.5^2 + 17 \times 3.5 + 3.5^2)}$ 的值。

2. If $a \# b = ab - 1$, find the value of $\frac{(2\#3)\#5}{2\#(3\#5)}$.

若 $a \# b = ab - 1$, 求 $\frac{(2\#3)\#5}{2\#(3\#5)}$ 的值。

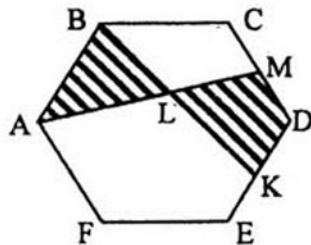
3. Let $x = \frac{1}{2} \left(\sqrt[3]{7} - \frac{1}{\sqrt[3]{7}} \right)$, find the value of $(x + \sqrt{1+x^2})^3$.

設 $x = \frac{1}{2} \left(\sqrt[3]{7} - \frac{1}{\sqrt[3]{7}} \right)$, 求 $(x + \sqrt{1+x^2})^3$ 的值。

4. Evaluate $1996 \times 19981998 - 1998 \times 19961995$.

求 $1996 \times 19981998 - 1998 \times 19961995$ 的值。

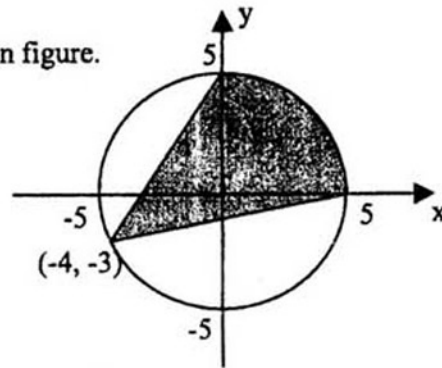
5. Given ABCDEF is a regular hexagon. M and K are the mid-points of CD and DE respectively, L is the point of intersection of AM and BK. If the area of $\triangle ABL$ is 16 cm^2 , find the area of the quadrilateral MLKD.



圖中 ABCDEF 爲一正六邊形。M, K 分別是 CD 與 DE 的中點，AM 與 BK 相交於 L。若 $\triangle ABL$ 的面積爲 16 cm^2 ，求四邊形 MLKD 的面積。

6. Find the area of the shaded region in the given figure.

求圖中陰影部分的面積。



7. The length of the sides of a triangle are 11, 15 and n , where n is a positive integer. For how many values of n are the triangle obtuse?

已知一個三角形的邊長為 11, 15 及 n , 其中 n 為正整數。問有多少個 n 的數值能令這個三角形成為鈍角三角形。

8. If 1998 is written as a product of two positive integers such that the difference of these two integers is as small as possible, find this difference.

若把 1998 寫成兩個正整數的積，並要求這兩個正整數之差儘量小，求這兩個正整數之差。

9. For how many values of n will an n -sided regular polygon have interior angles with integral degree measures?

問有多少個正 n 邊形的內角的角度是整數。

10. For any integer n , $E(n)$ denotes the sum of the odd digits of n . For example, $E(1998) = 1 + 9 + 9 = 19$. Find $E(1) + E(2) + \dots + E(98) + E(99)$.

對任意整數 n , 設 $E(n)$ 為組成整數 n 的數字中的單數的和。例如, $E(1998) = 1 + 9 + 9 = 19$ 。求 $E(1) + E(2) + \dots + E(98) + E(99)$ 。

11. Nine congruent spheres are packed inside a unit cube in such a way that one of them has its centre at the centre of the cube and each of the remaining 8 spheres touches the centre sphere and three faces of the cube. Find the radius of each sphere.

9 個全等的圓球體剛巧可以全部放進一個邊長為一個單位的正立方體內。其中一個球的球心位於立方體的中心。而餘下 8 個球則分別與正立方體其中的三個面及中心球體緊貼。求每一個球體的半徑。

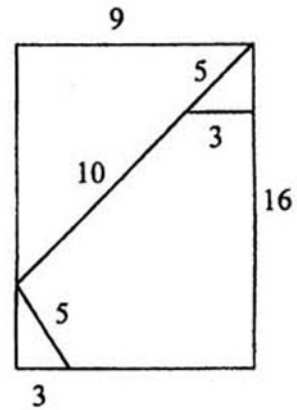
12. How many pairs (m, n) where m and n are positive integers are the solutions of the

$$\text{equation } \frac{4}{m} + \frac{2}{n} = 1?$$

方程式 $\frac{4}{m} + \frac{2}{n} = 1$ 有多少對正整數解 m, n ?

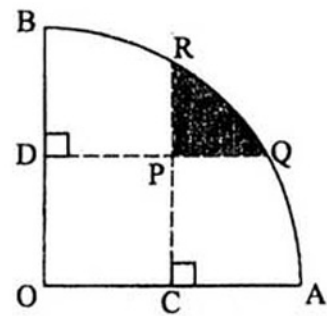
13. A rectangle, 16 by 9, is cut as in the manner shown. The pieces can then form a square. What is the perimeter of the square?

依圖中所示，一矩形，長 16，闊 9，被分成四塊後，可拼出另一正方形。求該正方形的周界。



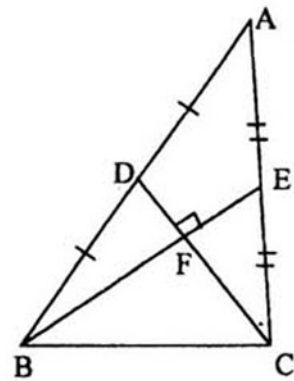
14. OAB is a quarter of a circle with radius 2 cm. C and D are the midpoints of OA and OB respectively. Find the area of the shaded region PQR .

OAB 是一個半徑為 2 cm 的 $\frac{1}{4}$ 圓。 C 及 D 分別為 OA 及 OB 的中點，求陰影部分 PQR 的面積。



15. In $\triangle ABC$, medians BE and CD are perpendicular to each other. If $BE = 7$, $CD = 6$, find the area of $\triangle ABC$.

$\triangle ABC$ 中，中線 BE 和 CD 互相垂直。若 $BE = 7$, $CD = 6$, 求 $\triangle ABC$ 的面積。



Part B (乙部)

Write the answers on the spaces provided in the answer sheet, you should also write down the working steps clearly.

請將答案填寫在答案紙上指定的位置，計算步驟必須清楚列出。

16. In a football tournament eight teams play against each other once, with two points awarded for a win, one point for a draw and zero for a loss. How many points must a team score to ensure that it is in the top four?

有八隊隊伍參加某足球比賽，賽事採單循環制，勝方得 2 分，賽和各得 1 分，負方得 0 分。問要晉身前四名，最少要取得多少分？

17. A $9 \times 9 \times 9$ cube is composed of twenty-seven $3 \times 3 \times 3$ cubes. The big cube is “tunnelled” as follows:

First, the six $3 \times 3 \times 3$ cubes which make up the centre of each face of the $9 \times 9 \times 9$ cube as well as the centre $3 \times 3 \times 3$ cube are removed.

Second, each of the twenty remaining $3 \times 3 \times 3$ cubes is “tunnelled” in the same way, that is, the centre facial unit cubes as well as each centre cube are removed.

Find the total surface area of the final figure.

27 個邊長為 3 的正立方體組成一個邊長為 9 的大正立方體。現在把大正立方體用以下程序「挖空」：

1. 移去大正立方體每面中央的六個小正立方體，以及大正立方體內正中央的小正立方體；
2. 把剩下的二十個小正立方體用同樣的方法再「挖空」；即移去每面中央邊長為 1 的正立方體及正中央的正立方體。

試計算餘下的立體的總表面面積。