

第二十八屆香港青少年數學精英選拔賽

**The 28th Hong Kong Mathematical
High Achievers Selection Contest**

2025 – 2026 (31 / 1 / 2026)

試題 Question Paper

甲部 (每題 2 分)

Part A (2 marks for each question)

把答案填在答題紙所提供的位置。

Write your answers in the spaces provided on the answer sheets.

1. 某街道兩旁建有相等數量的大廈，每棟大廈都與對面的大廈相對。由左方的大廈開始由 1 順序編號，即 1、2、3、.....，直到街尾最後一棟大廈，然後由右方街尾最後的一棟大廈繼續順序編號，直到右方的第一棟大廈為止。若編號 501 的大廈對面是編號 2026 的大廈，那麼，編號 2001 的大廈對面的大廈是甚麼編號？

On both sides of a street there are the same number of buildings, and each building directly faces a building on the opposite side. Starting from the leftmost building, the buildings are numbered consecutively 1, 2, 3, ... up to the last building at the far end of the street; the numbering then continues from that far end on the right-hand side back toward the first building on the right. If the building numbered 501 faces the building numbered 2026, what is the number of the building that faces building numbered 2001?

2. 求 2026^{2026} 可以被多少個正整數整除。

Find the number of positive factors of 2026^{2026} .

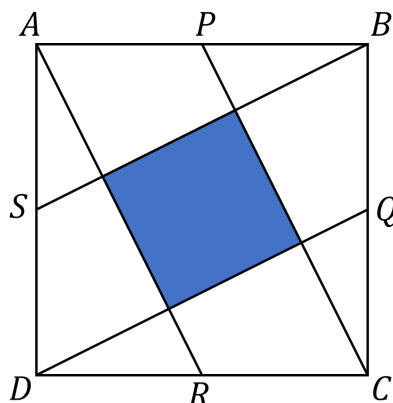
3. 有多少組正整數 (w, x, y, z) 滿足方程 $w + x + y + z = 10$?

How many sets of positive integers (w, x, y, z) satisfy the equation $w + x + y + z = 10$?

4. 下圖中 $ABCD$ 為正方形。 P, Q, R, S 分別為 AB, BC, CD, DA 的中點。 已知陰影區域面積為 16。 求 $ABCD$ 的面積。

In the figure below, $ABCD$ is a square. P, Q, R, S are the midpoints of AB, BC, CD, DA respectively.

Suppose the area of the shaded region is 16. Find the area of $ABCD$.



5. 若整數 A 是由整數 1 至 2026 的 2026 個數依次排列而成，即

$$A = 123456789101112131415\dots20252026,$$

那麼 A 是一個多少位的整數？

Let A be the integer obtained by forming the integers from 1 to 2026 in order, that is,

$$A = 123456789101112131415\dots20252026$$

How many digits does A have?

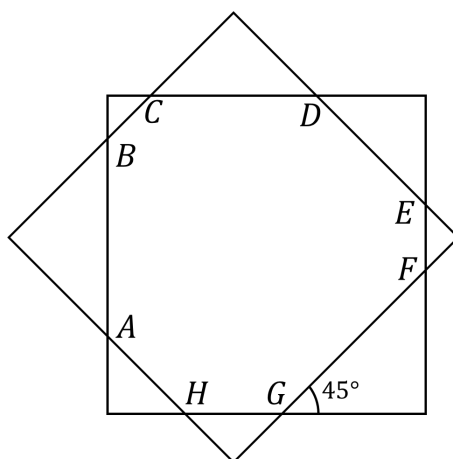
6. 若 $\frac{14231}{x(x+1)} + \frac{14231}{(x+1)(x+2)} + \frac{14231}{(x+2)(x+3)} + \dots + \frac{14231}{(x+2025)(x+2026)} = 2026$ 及 $x > 0$ ，求 x 的值。

If $\frac{14231}{x(x+1)} + \frac{14231}{(x+1)(x+2)} + \frac{14231}{(x+2)(x+3)} + \dots + \frac{14231}{(x+2025)(x+2026)} = 2026$ and $x > 0$, find the value of x .

7. 以下圖形由兩個全等正方形構成，且相交於八點。每一交點上的銳角皆為 45° 。

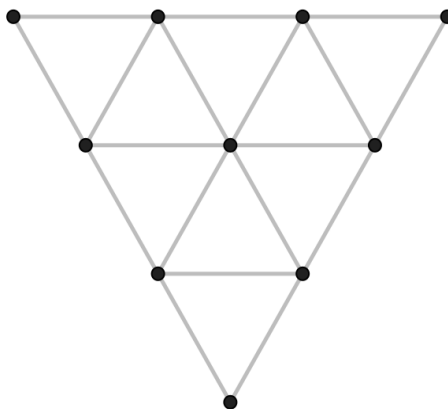
已知 $AB = 8, CD = 7, GH = 5$ ，求 EF 之長。

The figure below is formed by two congruent squares intersecting at eight points. At each intersecting point, the acute angle is 45° . Given that $AB = 8, CD = 7, GH = 5$. Find the length of EF .



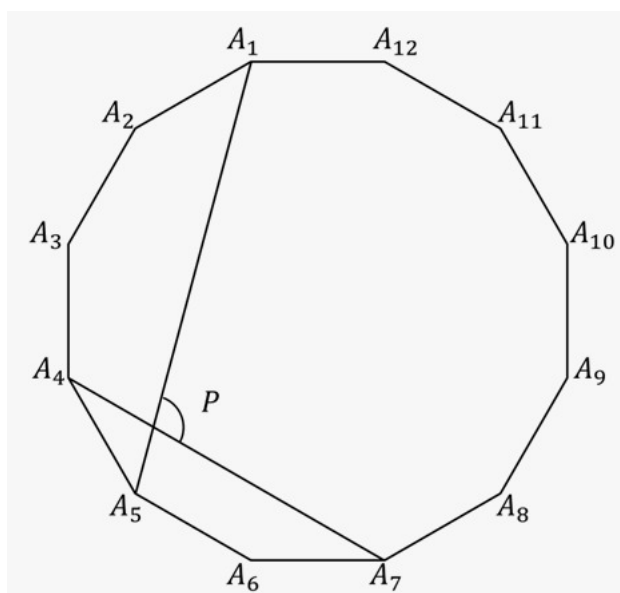
8. 下圖顯示是由等邊三角形構成的網格。若從這些點中隨機選取三個點，求它們形成一個等邊三角形的概率。

The figure below shows grids of equilateral triangles. If 3 points are randomly selected, find the probability that they form an equilateral triangle.



9. 若正整數 $(9n + 2)$ 能被正整數 $(45 - n)$ 整除，且 n 為正整數，求 n 的最小值。
Suppose positive integer $(9n + 2)$ is divisible by positive integer $(45 - n)$, where n is a positive integer. Find the minimum value of n .
10. $A_1 \dots A_{12}$ 為正十二邊形。 P 為兩條對角線 A_1A_5 及 A_4A_7 的交點，如圖所示。求 $\angle A_1PA_7$ 並以角度表達。

$A_1 \dots A_{12}$ is a regular dodecagon. P is the intersecting point of two diagonals A_1A_5 and A_4A_7 as shown in the figure. Find the angle $\angle A_1PA_7$ in degree.



11. 在下圖所示的每一個空格都填寫一個數字，使得任意三個相鄰格子中所填寫數字之和是 2026。
求 $a + b + c$ 的值。

Each cell in the figure below is filled with a number such that the sum of the numbers in any three consecutive cells is 2026. Find the value of $a + b + c$.

a			20				b				26					c
-----	--	--	----	--	--	--	-----	--	--	--	----	--	--	--	--	-----

12. a 和 b 均為正整數。已知 a 與 b 的積比 a 與 b 的和多 2025，求 $a + b$ 的最小值。
Let a and b be positive integers. It is known that the product of a and b exceeds their sum by 2025. Find the minimum value of $a + b$.

13. 已知 $\triangle ABC$ 中， P 、 Q 、 R 分別為 BC 、 CA 及 AB 上的點， AP 、 BQ 、 CR 相交於 O 。

若 $\frac{AO}{OP} + \frac{BO}{OQ} + \frac{CO}{OR} = 10$ ，求 $(\frac{AO}{OP})(\frac{BO}{OQ})(\frac{CO}{OR})$ 的值。

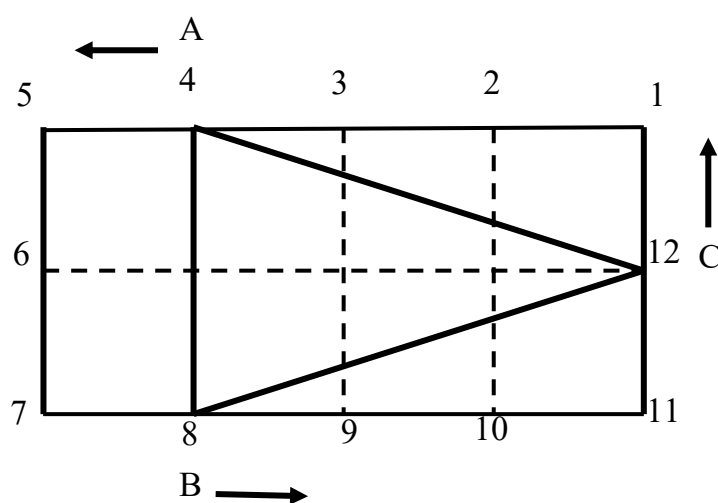
$\triangle ABC$ is a triangle. P , Q and R are points on BC , CA and AB respectively such that AP , BQ , CR are concurrent and meet at a point O . If $\frac{AO}{OP} + \frac{BO}{OQ} + \frac{CO}{OR} = 10$, find the value of $(\frac{AO}{OP})(\frac{BO}{OQ})(\frac{CO}{OR})$.

14. 一個盒子裡有紅、黃、藍和白色波子各 26 粒，若小明被蒙上眼睛從該盒子中抽出波子，問至少要抽多少粒波子才確保抽到 20 粒顏色相同的波子？

A box contains 26 red, 26 yellow, 26 blue, and 26 white marbles. If Siu Ming draws marbles from the box while blindfolded, what is the minimum number of marbles he must draw to guarantee that he has at least 20 marbles of the same colour?

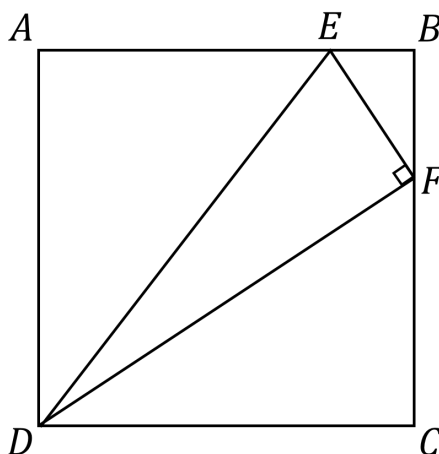
15. 如圖，在一個長 4 cm、闊 2 cm 的長方形周邊，每隔 1 cm 編一個編號，從 1 號至 12 號。最初， $\triangle ABC$ 的三個頂點 A 、 B 及 C 分別位於 4、8 及 12 的編號上，以反時針方向沿長方形的周邊每次移動 1 cm，即三個頂點的編號變成 5、9 及 1 號，並組成另一個三角形。如是者，三個頂點分別繞了該長方形一周，在各自回到原先的位置前， $\triangle ABC$ 有多少次組成一個直角三角形？

Consider a rectangle with length 4 cm and width 2 cm as shown in the figure. Points along its perimeter are numbered at every 1 cm from 1 to 12. Initially, the vertices A, B, C of $\triangle ABC$ are at positions 4, 8, and 12, respectively. The vertices move anti-clockwise along the perimeter 1 cm at a time, so that after one move, their positions become 5, 9, and 1, forming another triangle. Before the vertices return to their original positions after completing one full lap around the rectangle, how many times does $\triangle ABC$ form a right-angled triangle?



16. $ABCD$ 為正方形。 E, F 分別為 AB, BC 上之兩點。 設 $EF = 1, DF = 4$ 且 $\angle EFD = 90^\circ$ 。 求正方形 $ABCD$ 之邊長。

$ABCD$ is a square. E, F lie on AB, BC respectively. Suppose $EF = 1, DF = 4$ and $\angle EFD = 90^\circ$. Find the side length of square $ABCD$.



17. 如下圖所示，紙張 ABC 中，已知 $AC = CB$ 及 $\angle ACB = 90^\circ$ 。進行以下摺紙操作：

(i) 摺痕 BD 平分 $\angle ABC$ 然後打開；

(ii) 沿 KL 將點 B 摺到點 D ；

(iii) 沿 DN 將點 A 摺到點 K 。

此時形成了五邊形 $NDCLK$ 。求 $\angle NKL$ 。

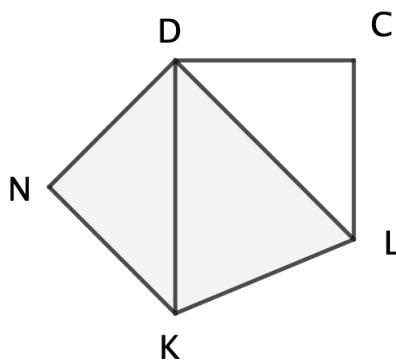
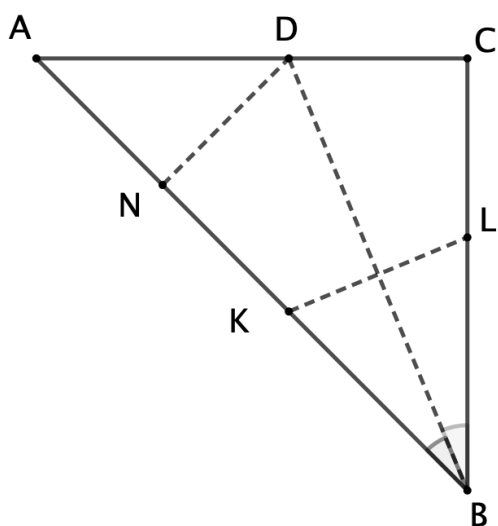
A triangular paper ABC is shown in the figure below. Given that $AC = CB$ and $\angle ACB = 90^\circ$. The following folds are performed:

(i) Fold along BD which bisects $\angle ABC$ and then unfold;

(ii) Fold along KL such that B & D coincide;

(iii) Fold along DN such that A & K coincide.

Then a pentagon $NDCLK$ is formed. Find $\angle NKL$.



18. 求 3^{2026} 的最尾兩位數字。

Find the last two digits of 3^{2026} .

乙部 (每題 6 分)

Part B (6 marks for each question)

把完整的題解和答案寫在答題紙所提供的位置。

Write your full solution and answers in the spaces provided on the answer sheets.

19. 有多少個兩位正整數 n ，使得 n 能被它的十位數及個位數整除？

How many two-digit positive integers n are there such that n is divisible by the ten-digit and the unit-digit?

20. 六名學生 A、B、C、D、E 和 F 進行了一次測驗，得到不同的分數。以下是他們的對話。

A 說：「我的分數比 C、E 和 F 的分數高。」

B 說：「我的分數比 A、E 和 F 的分數高。」

C 說：「我的分數比 A 和 E 當中恰好一人的分數高。」

D 說：「我的分數比 B 的分數高。」

E 說：「我的分數比 A、B 和 F 當中恰好兩人的分數高。」

F 說：「我的分數比 B、C 和 E 當中恰好兩人的分數高。」

已知恰好一名學生在說謊。試將這些學生按照他們的分數從高至低排列。

Six students A, B, C, D, E and F take a test and obtain different scores. The following is their conversation.

A says, 'my score is higher than the scores of C, E and F.'

B says, 'my score is higher than the scores of A, E and F.'

C says, 'my score is higher than the scores of exactly one of A and E.'

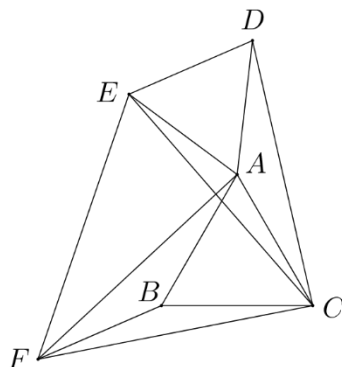
D says, 'my score is higher than the score of B.'

E says, 'my score is higher than the scores of exactly two of A, B and F.'

F says, 'my score is higher than the scores of exactly two of B, C and E.'

It is known that exactly one of the students is lying. Arrange the scores of these students in descending order.

21. 如圖所示， $\triangle ABC$ 和 $\triangle ADE$ 都是等邊三角形。假設 $AF = CD$ 及 $BF = AD$ 。試證明 $\triangle CEF$ 是等邊三角形。As shown in the figure, $\triangle ABC$ and $\triangle ADE$ are equilateral triangles. Suppose $AF = CD$ and $BF = AD$. Show that $\triangle CEF$ is an equilateral triangle.



~ 全卷完 End of Paper ~