第廿五屆香港青少年數學精英選拔賽

The 25th Hong Kong Mathematical High Achievers Selection Contest

2022 – 2023（4 / 2 / 2023）

試題 Question Paper

甲部　　(每題 2 分)

把答案填在答題紙所提供的位置。

1. 35 個自然數的平均數為 x (準確至三位小數)。小明準確計算出這平均數卻將最後一個位寫錯了。他的錯誤答案是 2.023，求 x 的值(準確至三位小數)。
The mean of 35 natural numbers is x (correct to 3 decimal places). Ming calculated the mean correctly but he miswrote the last digit. His incorrect answer is 2.023. Find the value of x (correct to 3 decimal places).

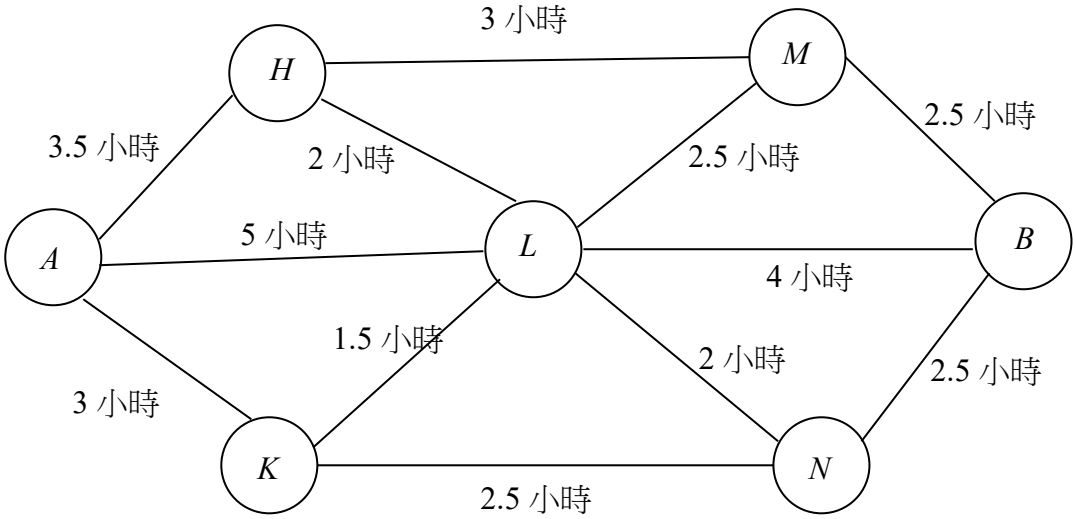
2. 小明、小強兩人合做某件工作，需要 10 小時完成。假如他們一起工作了 4 小時後，小明先離去，那麼小強便需要 18 小時獨力完成餘下工作。
求小明單獨完成整件工作所需的時間。
Ming and Keung finish a task together in 10 hours. If they work together for 4 hours and Ming leaves afterwards, then Keung has to spend 18 hours to finish the remaining task on his own.
Find the working time Ming needs to finish the whole task on his own.

3. 求 $2023-\sqrt{1+2023\sqrt{1+2022\sqrt{1+2021\times 2019}}}$ 的值。
Find the value of $2023-\sqrt{1+2023\sqrt{1+2022\sqrt{1+2021\times 2019}}}$.

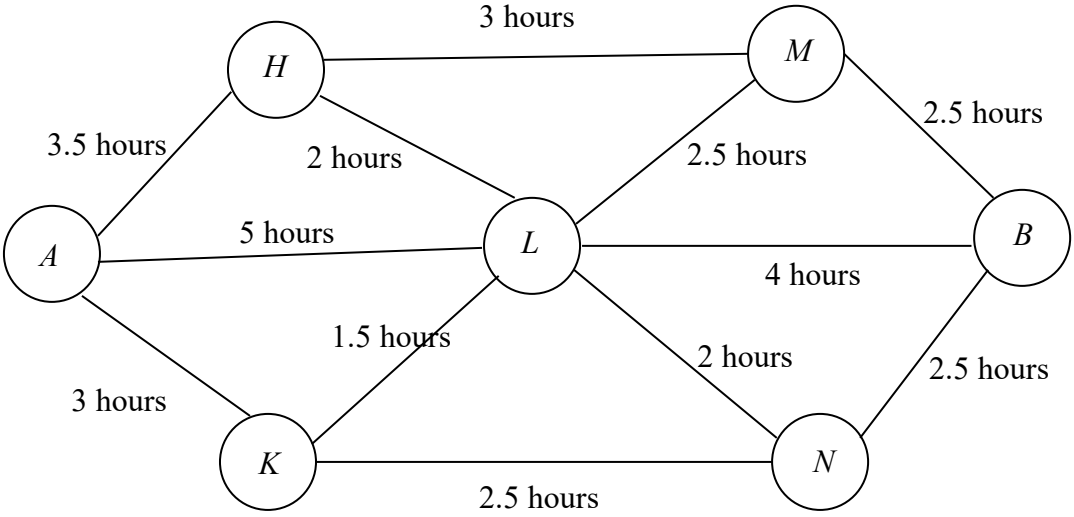
4. 若 $\sqrt{5\sqrt{n-3}-n+1}$ 及 n 均為整數，求 $\sqrt{5\sqrt{n-3}-n+1}$ 的最少可能值。
If both $\sqrt{5\sqrt{n-3}-n+1}$ and n are integers, Find the least possible value of $\sqrt{5\sqrt{n-3}-n+1}$.

5. 已知實數 a、b、c 滿足 $3a+12b+2c=63$ 及 $2a+7b+c=35$。求 $\frac{a+2b}{3b+c}$ 的值。
It is known that real numbers a, b and c satisfy $3a + 12b + 2c = 63$ and $2a + 7b + c = 35$. Find the value of $\frac{a+2b}{3b+c}$.

6. 下圖為城市 *A* 和城市 *B* 之間的路線圖，當中包括五個市鎮：*H*、*K*、*L*、*M* 及 *N*，而路線上的數字則代表由一個市鎮/城市前往另一個市鎮/城市的所需時間。若小明需要由城市 *A* 前往城市 *B*，試找出所需的最短時間。



The following figure shows the routes between City *A* and City *B*. There are 5 towns *H*, *K*, *L*, *M* and *N* between the two cities. The numbers beside the straight lines represent the time required to travel from a city/town to another. Find the shortest time required for Ming to travel from City *A* to City *B*.

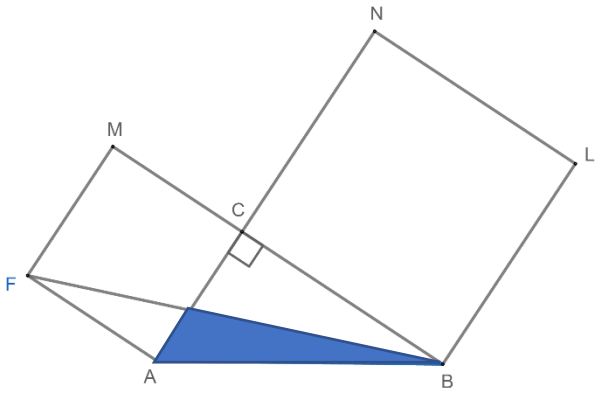


7. *ABCDEFGH* 是一正立方體。它有 12 條棱和 6 個面，而每個面有兩條對角線，因此共有 12 條棱和 12 條對角線，總共 24 條線段。已知從這 24 條線段中可以選出 *N* 條，使得其中任何兩條線段均非共面，求 *N* 的最大值。

ABCDEFGH is a cube. It has 12 edges and 6 faces. On each face there are 2 diagonals. Altogether there are 12 edges and 12 diagonals, which means a total of 24 line segments. It is known that out of these 24 line segments, one can choose *N* of them such that no two of these *N* line segments are co-planar. What is the maximum value of *N*?

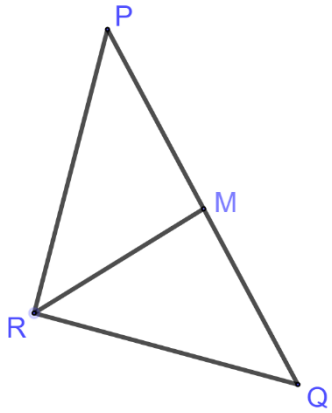
8. ΔABC 為直角三角形，其中 $AC = 3$、$BC = 4$ 及$\angle ACB = 90^\circ$。$ACMF$ 與 $BLNC$ 皆為正方形。求陰影面積。

ΔABC is a right-angled triangle where $AC = 3$, $BC = 4$ and $\angle ACB = 90^\circ$. $ACMF$ and $BLNC$ are squares. Find the shaded area.



9. ΔPQR 的周界為 48，M 為 PQ 的中點，且 PM=RM=10，求ΔPQR 的面積。

The perimeter of ΔPQR is 48, M is the mid-point of PQ and PM=RM=10, find the area of ΔPQR.



10. 在八進制的 9 位數中能被 7 整除且各位數均為 0 或 4 的數有多少個？

How many 9-digit octal numbers which are divisible by 7 and all digits are 0 or 4?

11. 一個花圃被分成六個區域(如圖)。現要栽種四種不同顏色的玫瑰花，每個區域栽種一種且相鄰區域不能栽種相同顏色的玫瑰花。請問有多少種栽種方法?

A garden is divided into 6 regions as shown in the diagram. Roses of 4 different colours will be grown in the garden. Each region will accommodate roses of one colour and neighbouring regions will have roses of different colours. In how many ways can roses be grown in the garden?

<table>
<tr><td rowspan="2">1</td><td>2</td><td rowspan="2">4</td></tr>
<tr><td>3</td></tr>
<tr><td colspan="2">5</td><td>6</td></tr>
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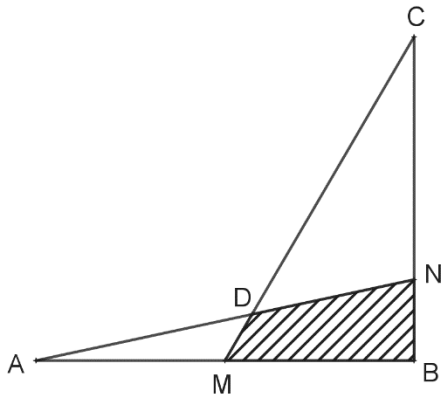
12. 已知 ABC 為三角形，其中 $\angle ABC = 90^o$，$BC = 4AB$。現將 BA 延長至 D，使 $BA = 7AD$。E 是由 B 至 AC 的垂足，同時 BE 延線交 CD 於 F。求 CF: FD。
It is given that ABC is a triangle with $\angle ABC = 90^o$, and $BC = 4AB$. BA is produced to D so that $BA = 7AD$. E is the foot of perpendicular from B to AC, and BE is produced to meet CD at F. Find the ratio CF: FD.

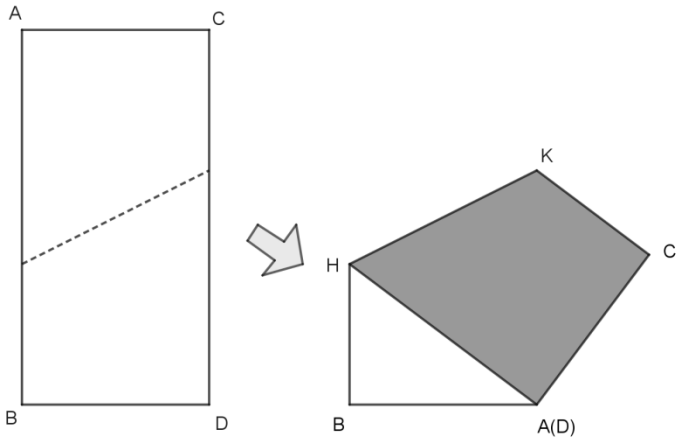
13. $ABCD$ 是一個正方形，每一邊的長度為 4。E 及 F 分別為 AD 及 CD 上的點，使 AE =1 及 CF = 2。P 是 EF 上的一點，而 Q 及 R 分別為 P 至 BC 及 AB 的垂足。求矩形 $BQPR$ 的最大面積。
$ABCD$ is a square with each side equal to 4. E and F are points on AD and CD respectively such that AE =1 and CF = 2. P is a point on EF, and Q, R are the feet of perpendicular from P to BC and AB respectively. What is the maximum possible area of the rectangle $BQPR$?

14. 求$2^{3^{11}}$的個位數。
Find the unit digit of $2^{3^{11}}$.

15. 下圖中，AMB 及 BNC 為直線，四邊形 $ABCD$ 的面積等於 48 cm^2。已知 AM=MB 及 BN : NC = 1 : 3，求四邊形 $MBND$ 的面積。
The area of quadrilateral $ABCD$ shown below is 48 cm^2 where AMB and BNC are straight lines. It is given that AM=MB and BN : NC = 1 : 3, find the area of quadrilateral $MBND$.



16. 如下圖，把矩形紙 *ABDC* 摺疊起來，使 *A* 點和 *D* 點重合後得到五邊形 *BDCKH*。如果 *AB* =12 cm 及 *BD* = 6 cm，求五邊形 *BDCKH* 的面積。
The rectangular *ABDC* shown below is folded such that point *A* and point *D* overlap to obtain a pentagon *BDCKH*. If *AB* =12 cm and *BD* = 6 cm, find the area of *BDCKH*.



17. 2023 年 1 月 1 日是星期日。下一次 1 月 1 日是星期日會是哪一年呢?
January 1, 2023 is Sunday. In which coming year will January 1 be Sunday again?

18. 1223334444…是一個以 1 個 1，緊接 2 個 2，緊接 3 個 3，緊接 4 個 4 如此類推的一個數。若它有 2023 個位，最後的那個數字是甚麼？
The number 1223334444… is written down by starting with one 1, then two 2s, then three 3s, then four 4s, and so on. If it has 2023 digits, what is the last digit?

乙部　　(每題 6 分)

把完整的題解和答案寫在答題紙所提供的位置。

19. 某人在寫下兩個兩位數的乘積時，意外地遺漏了中間的乘號，結果變成了一個四位數。若該四位數等於原來乘積的 2 倍，求該四位數。
When someone wrote down the product of two two-digit numbers, the multiplication sign in between was accidentally omitted, resulting in a four-digit number. If the four-digit number is equal to 2 times the original product, find the four-digit number.

20. 從正方形、正 m 邊形及正 n 邊形分別各取一隻內角，在沒有重疊的情況下，該三隻角可恰好拼成一隻周角($m \le n$)。求所有可能的序偶(m, n)。
Take an interior angle from a square, a regular m-sided polygon and a regular n-sided polygon respectively. Without overlapping, these three angles can form a round angle($m \le n$). Find all possible order pairs (m, n).

21. N 是一個大於 1 的正整數。N 的兩倍減 1 可得一數，它由 N 原來的數字倒序而成。
(a) N 的首個數字必定是甚麼？
(b) N 的最末數字必定是甚麼？
(c) N 必定是甚麼？求 N 的所有可能值。
N is a positive integer larger than 1. Double N and subtract 1 from the result we obtain a number that is formed from the original number N by reversing its digits.
(a) What must the first digit of N be?
(b) What must the last digit of N be?
(c) What must N be? That is, find all such numbers N.

~ 全卷完 End of paper ~

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