

香港青少年數學精英選拔賽

The Hong Kong Mathematical High Achievers Selection Contest

2018 – 2019

甲部 (每題 2 分) 把答案填在答題紙所提供的位置。

- 1 1 至 2019 中，有多少個數是 3 的倍數且與 15 互質？

From 1 to 2019, how many numbers are multiples of 3 and are relatively prime with 15?

- 2 已知 a^{b^c} 表示 a 的 b^c 次方，求 $2017^{2018^{2019}}$ 的個位數。

It is given that a^{b^c} represents a raised to the power b^c . Find the last digit of $2017^{2018^{2019}}$.

- 3 $2019 \times 2018 \times 2017 \times \dots \times 1$ 的最右邊有多少個 0？

How many 0's are there at the right end of $2019 \times 2018 \times 2017 \times \dots \times 1$?

- 4 求 2019^{2019} 可以被多少個正整數整除。

Find the number of positive integers which can divide 2019^{2019} .

- 5 一盒波子裡有紅、黃、藍、白和黑色各 2019 粒，若甲隨機抽出波子，問至少要抽多少粒波子才能確保必定抽到 64 粒相同的波子？

There are 2019 pieces of red, yellow, blue, white and black marbles each. If Peter takes out the marbles randomly, at least how many marbles need to be taken to ensure that there will be 64 identical marbles?

- 6 若 $\frac{3}{m} + \frac{6}{n} = 1$ ，其中 m 、 n 為正整數，求 $m + n$ 的最大值。

If $\frac{3}{m} + \frac{6}{n} = 1$, where m, n are positive integers, find the maximum value of $m + n$.

- 7 有 20 個不同正整數，從大至小排列。若該 20 個數的平均值是 19，求第五個數的最大值。

There are 20 different positive integers arranged in descending order. If the mean of these 20 numbers is 19, find the greatest possible value of the fifth number.

- 8 x 及 y 均為正整數，求 $2(3xy - 19)$ 及 $(6xy + 20)$ 這兩數之間(包括首尾兩數)有多少個偶數？

x and y are positive integers. How many even numbers are there between $2(3xy - 19)$ and $(6xy + 20)$ inclusively?

- 9 把 2019 人分成四隊。第一隊人數是第二隊的 $1\frac{1}{3}$ 倍，也是第三隊的 $1\frac{1}{4}$ 倍，若第四隊有 N 人，求 N 的最小值。

2019 people are divided into 4 teams. The number of people in the first team is $1\frac{1}{3}$ times of that in the second team and also $1\frac{1}{4}$ times of that in the third team. If there are N people in the fourth team, find the minimum value of N .

- 10 導遊將五間房間分配給五位團友，然後再隨意將五條房匙分配給他們，已知僅有三人取得正確的房門匙，求有多少種不同分配房門匙的方法？

A guide distributes five rooms to five customers. Then the guide distributes the five room keys to the five customers randomly. It is known that exactly three of the customers get their room keys correctly. How many different ways of distributing the keys to the five customers are there?

- 11 已知等腰三角形 XYZ 的面積為 48，若兩腰長之和等如底邊長與底邊上的高之和，求三角形 XYZ 的周界。

It is given that the area of an isosceles triangle XYZ is 48. If the sum of the length of its two equal sides is equal to the sum of its base and the corresponding height, find the perimeter of triangle XYZ .

- 12 使用 1 至 9 九個數字各一次，形成 3 個三位數，考慮最大數及最小數之差。(例如，若該 3 個數為 123、456 及 789，則差為 $789 - 123 = 666$ 。)
求該差的最小可能值。

Each of the nine digits from 1 to 9 is used once to form 3 three-digit numbers.

Consider the difference between the largest and the smallest numbers. (For example, if the 3 numbers formed are 123, 456 and 789, then the difference is $789 - 123 = 666$.) Find the least possible value of this difference.

- 13 一個六位數 $\overline{a2019b}$ 能被 24 整除，其中 a 和 b 是個位數，求該六位數的最大值。

A six-digit number $\overline{a2019b}$ can be divided by 24, where a and b are single digits. Find the maximum value of the six-digit number.

- 14 2019 個連續偶數的和是 $2019N^2$ ，其中 N 為正整數。求 N 的最小值。

The sum of 2019 consecutive even numbers is $2019N^2$ where N is a positive integer. Find the minimum value of N .

- 15 設 $a_1 = 2019$ ， $a_2 = 1999$ ，而對所有正整數 n ， $a_{n+2} = a_{n+1} - a_n$ 。
求 $a_1 + a_2 + a_3 + \dots + a_{2019}$ 的值。
Let $a_1 = 2019$, $a_2 = 1999$, and $a_{n+2} = a_{n+1} - a_n$ for all positive integers n .
Find the value of $a_1 + a_2 + a_3 + \dots + a_{2019}$.
- 16 兩數之差、和與積之比為 $1 : 9 : 40$ ，求該兩數的積。
If the ratio of the difference, sum and product of two numbers is $1 : 9 : 40$, find the value of the product of these two numbers.
- 17 已知 a 及 b 為質數而 c 為奇數，若 $a^2 + b^2 c^2 = 1229$ ，求 $b + c - a$ 的值。
It is given that a and b are prime numbers while c is an odd number. If $a^2 + b^2 c^2 = 1229$, find the value of $b + c - a$.
- 18 在 2019 張卡紙上分別寫上 $1, 2, \dots, 2018, 2019$ ，然後將該 2019 張卡紙放入 673 個盒內。若該些盒內卡紙上數字之總和皆不相同，其中最小為 N ，求 N 的最大值。
There are 2019 cards with number $1, 2, \dots, 2018, 2019$ and these cards will be put into 673 boxes. If the sums of the numbers on the cards in those boxes are all different, where the minimum is N , find the maximum value of N .

乙部 (每題 6 分)

把完整的題解和答案寫在答題紙所提供的位置。

- 19 就第十九題，徵詢專家顧問意見後，認為該題屬研究性問題，不宜作數學競賽之用。為恪守嚴謹及公平原則，經慎重考慮，議決取消該題，敬希垂注。
- 20 附圖是由一個等邊三角形及三個相同的圓形組成。

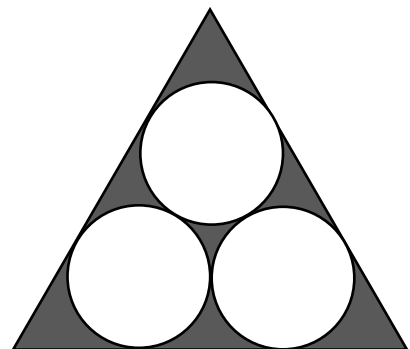
若陰影部分的面積為 $(16\sqrt{3} + 24 - 12\pi) \text{ cm}^2$ 。

求該三角形的周界。

The figure shown is formed by an equilateral triangle and three equal circles.

If the shaded area is $(16\sqrt{3} + 24 - 12\pi) \text{ cm}^2$.

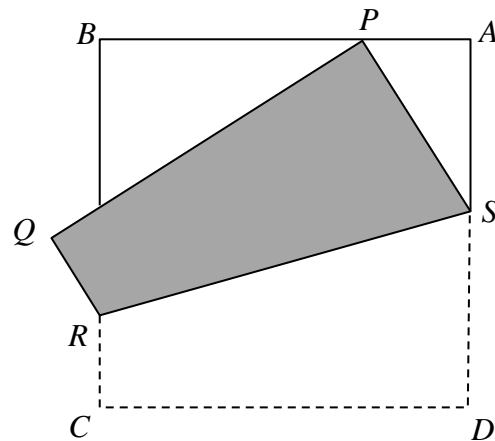
Find the perimeter of the triangle.



- 21 一張正方形紙 $ABCD$ 的正面為白色，而背面為灰色， $AB = 1$ cm。 R 及 S 分別為邊 BC 及 AD 上的變點。 把這張紙沿 RS 摺起使得角 D 落在 AB 上的一點 P ，形成一個灰色的四邊形 $PQRS$ ，如圖所示。
- (a) 若 $AP = x$ cm，以 x 表 RS 、 PS 及 QR 。
- (b) P 必須在何處才使得灰色的四邊形 $PQRS$ 有最大的面積？求該最大面積。
- (c) P 必須在何處才使得灰色的四邊形 $PQRS$ 有最小的面積？求該最小面積。

A piece of square paper $ABCD$ is white on the front side and grey on the back, and $AB = 1$ cm. R and S are two variable points on the sides BC and AD respectively. Fold up this piece of square paper along RS so that the corner D falls on AB at a point P , producing a grey quadrilateral $PQRS$, as shown in the figure.

- (a) If $AP = x$ cm, express RS , PS and QR in terms of x .
- (b) Where should P be so that the grey quadrilateral $PQRS$ has a maximal area? Find this maximal area.
- (c) Where should P be so that the grey quadrilateral $PQRS$ has a minimal area? Find this minimal area.



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