香港青少年數學精英選拔賽

The Hong Kong Mathematical High Achievers Selection Contest
2016 – 2017

甲部　**(每題 2 分)**
把答案填在答題紙所提供的位置。

1 計算 $2018 \times 20172017 - 2017 \times 20182016$。
　 Calculate　$2018 \times 20172017 - 2017 \times 20182016$.

2 求 2017^{2016} 的個位數。
　 Find the last digit of 2017^{2016}.

3 計算 $\dfrac{(2017^2 - 2 \times 2017 - 8)(2017^2 - 2017 - 2)}{2013 \times 2015 \times 2016 \times 2018 \times 2019}$。

　 Calculate $\dfrac{(2017^2 - 2 \times 2017 - 8)(2017^2 - 2017 - 2)}{2013 \times 2015 \times 2016 \times 2018 \times 2019}$.

4 圖中，*ABCD* 及 *BEFG* 為正方形，*BC* 及 *DE* 相交於 *H* 及 *AB* = 16。若 ΔAEH 的面積為 72，求 *EF* 的長度。
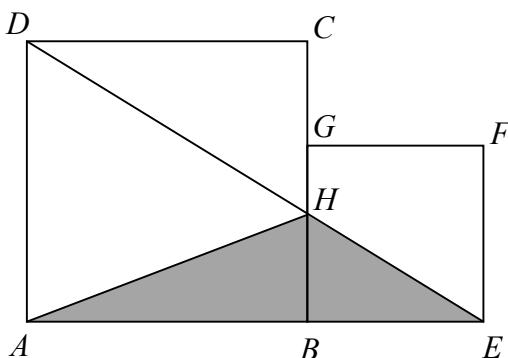

In the figure, *ABCD* and *BEFG* are squares, *BC* and *DE* intersect at *H* and *AB* = 16. If the area of ΔAEH is 72, find the length of *EF*.

5 用 {2、0、1、7} 四個數字，不允許重複，能寫出多少個不同的三位數？
　 Using the 4 digits {2, 0, 1, 7} without repetition, how many different 3-digit numbers can be formed?

6 若某年的 7 月有 4 個星期日，5 個星期六，問該年 7 月 1 日是星期幾？
　 In July of a certain year, there are 4 Sundays and 5 Saturdays, which day of week is 1st of July of that year?

7　圖中，長方形 *ABCD* 的長和闊的值都是整數。其中三角形 *ABD* 的周界的值是長方形 *ABCD* 面積的值的一半，若長方形 *ABCD* 的面積是 *S*，求 *S* 的最小值。
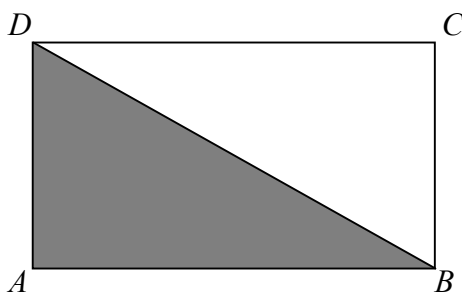
In the figure, both values of the length and the width of the rectangle *ABCD* are integers. The value of the perimeter of the triangle *ABD* is half that of the area of the rectangle *ABCD*. If the area of the rectangle *ABCD* is *S*, find the minimum value of *S*.

8　有 2016 個不相同的真分數，當中任何 2015 個的和化至最簡時，均是分母為 2017 的真分數。求該 2016 個真分數的和。

There are 2016 different proper fractions. The sum of any 2015 of them is a proper fraction with denominator 2017 when expressed in its simplest form. Find the sum of these 2016 proper fractions.

9　已知一個三位數的任意兩個數字之和等於第三個數字的倍數，問這樣的三位數有多少個？

It is given that the sum of any two digits of a 3-digit number is equal to a multiple of the remaining digit. How many such 3-digit numbers are there?

10　圖中，*ABC* 為一個三角形。
D 及 E 分別為 *AB* 及 *AC* 上的點使得 *AD* = *AE* = *DC* = *EB* = *BC*。　求 ∠*BAC*。
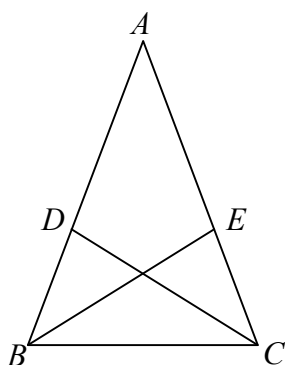
In the figure, *ABC* is a triangle. *D* and *E* are points on *AB* and *AC* respectively such that $AD = AE = DC = EB = BC$. Find ∠*BAC*.

11　對正整數 n，記 $1 \times 2 \times 3 \times \acute{\imath} \times n = n!$。若 $M = 1! \times 2! \times \acute{\imath} \times 10!$，求在 M 的正因子中為立方數的數目。

For positive integer n, denote $1 \times 2 \times 3 \times \acute{\imath} \times n = n!$. If $M = 1! \times 2! \times \acute{\imath} \times 10!$, find the number of positive factors of M whose are perfect cubes.

12　在一個高 20 cm 及底半徑為 17 cm 的圓錐體容器內加入一些水，並在容器的側面畫上水高度的標誌。

將容器上下倒置時，水沒有漏失，且水面正好是在剛才側面畫上水高度的標誌位置。設這時水的高度為 x cm，求 x 的值。

Add water to a conical vessel whose height and base radius are 20cm and 17cm respectively. The water level is marked.

When the vessel is inverted with no water loss, the water level is same as the level marked before. Suppose that the height of the water level is x cm, find the value of x.

20 cm

17 cm

x cm
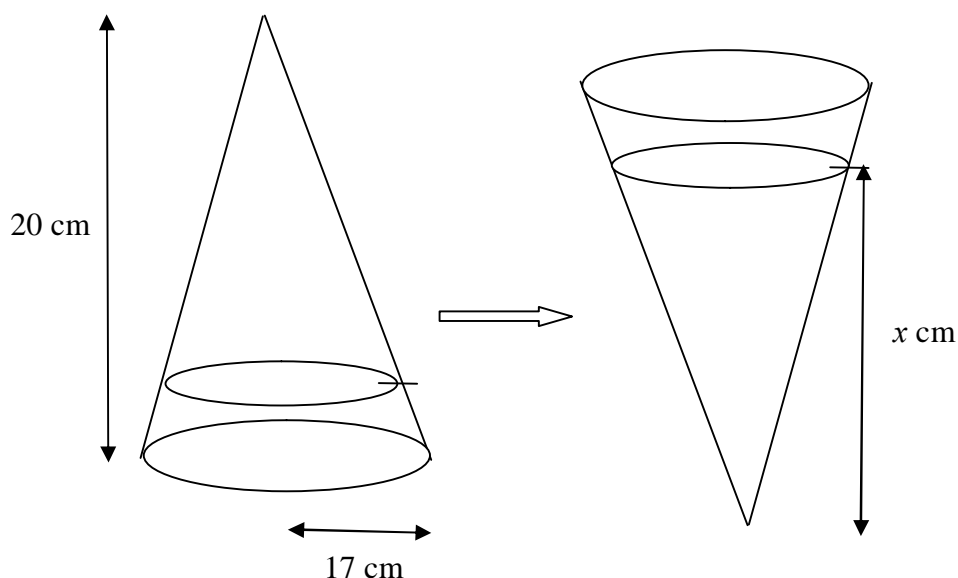
13　小明與小華從圓形跑道上對著的兩點 A 及 B 出發（即 AB 是圓的一條直徑），各自以不同但均勻的速度相反方向而行。一分鐘後兩人迎面相遇，然後立即各自繼續向前行，問再過多少分鐘後兩人又再次迎面相遇呢？

Ming and Wah start to run from two diametrically opposite points A and B on a circular track (i.e. AB is a diameter of the circle.), each with different but uniform speeds and in opposite directions. They meet after one minute and each continues to run on his course round the track immediately. How many minutes later will they meet again?


A　B
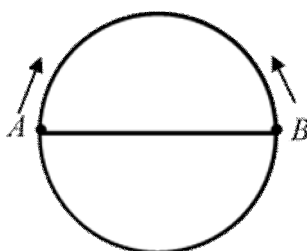
14 開始時有 3 個數 $\{1, 2, 3\}$，每次操作把其中一個數換成其餘兩個數之和。經過 7 次操作之後的 3 個數中，最大的數為 M。求 M 的最大可能值。

At the beginning, there are 3 numbers $\{1, 2, 3\}$. In each operation, one of the numbers is replaced by the sum of the other two numbers. After 7 operations, M is the greatest number of these 3 numbers. Find the greatest possible value of M.

15 在 ΔABC 中，D 為 AB 上的一點且 CD 平分 $\angle ACB$、$\angle A = 2\angle B$、$AC = 11$ 及 $AD = 2$。求 BC 的長度。

In ΔABC, D is a point on AB such that CD bisects $\angle ACB$, $\angle A = 2\angle B$, $AC = 11$ and $AD = 2$. Find the length of BC.
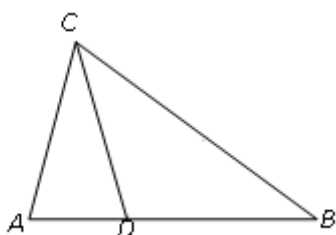
16 在三角形 ABC 中，$AB = 3$、$BC = 4$ 及 $AC = 5$。設 X 為三角形內的一點。求 $AX^2 + BX^2 + CX^2$ 的最小值。

In ΔABC, $AB = 3$, $BC = 4$ and $AC = 5$. Let X be a point inside the triangle. Find the least value of $AX^2 + BX^2 + CX^2$.

17 一個 3×3 的網格中的每個方格均填有一個數字，該數字代表對應的長方形的周界。 圖中，其中四個方格已填上已知的數字，其餘的五個方格的數字則以 p、q、r、s 及 t 代表。

(a) 求 p 的數值。

(b) 若 $q = 20$， 求 r、s 及 t 的數值。

10	t	16
s	10	r
16	q	p

Each cell in the 3×3 grid is marked with a number denoting the **perimeter** of the corresponding rectangle. In the figure, four of the cells are marked with the given numbers, while the numbers in the other five cells are denoted by p, q, r, s and t.

(a) Find the value of p.

(b) If $q = 20$, find the values of r, s and t.

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18   如圖，在菱形 *ABCD* 中，*E* 為 *BC* 上的點使得 *AE*⊥*BC*，$\cos B = \dfrac{4}{5}$ 及 *EC* = 2。*P* 為

AB 上的點。求 *PE* + *PC* 的最小值   。
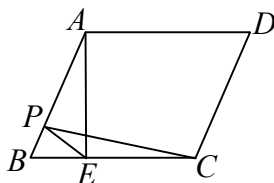
In the figure, *ABCD* is a rhombus. *E* is a point on *BC* such that $AE \perp BC$, $\cos B = \dfrac{4}{5}$

and *EC* = 2. *P* is a point on *AB*. Find the least value of *PE* + *PC*.

乙部　**(每題 6 分)**

把完整的題解和答案寫在答題紙所提供的位置。

19   求 $3^{2046} - 1$ 及 $3^{2016} - 1$ 的最大公因數。

Find the highest common factor of $3^{2046} - 1$ and $3^{2016} - 1$.

20   將 1、 2、3、…、49、50 這五十個數，任意分成 10 組，每組 5 個數，在每組中
依數值由細至大排列，記居中的數為「中位數」。設 S 為這 10 個「中位數」之和，
求 S 的

(a) 最大值；

(b) 最小值。

Divide the fifty numbers 1, 2, 3, í ., 50 into 10 groups ,with 5 numbers in each group.
Arrange the numbers in ascending order of magnitude in each of the group and the
number in the middle is called ˉmiddle numberø Suppose S is the sum of these 10
ˉmiddle numberø find

(a) the greatest value of S

(b) the smallest value of S.

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21　(a)　求任何四個正方形的面積，需滿足以下條件：

　　　　(1) 該四個正方形的邊長為整數；及

　　　　(2) 該四個正方形的面積為正數 A_1、A_2、B_1 及 B_2，其中

　　　　　　　$A_1 ó A_2 = B_1 ó B_2 > 0$　　及　　$A_1 > B_1$。

　　(b)　求任何六個正方形的面積，需滿足以下條件：

　　　　(1) 該六個正方形的邊長為整數；及

　　　　(2) 該六個正方形的面積為正數 A_1、A_2、A_3、B_1、B_2 及 B_3，其中

　　　　　　　$A_1 ó A_2 = B_1 ó B_2 > 0$　，　$A_2 ó A_3 = B_2 ó B_3 > 0$　及　$A_1 > B_1$。

(a)　Find the areas of any four squares satisfying the following conditions:

　　(1) the lengths of the sides of the four squares are **integers**; and

　　(2) the areas of the four squares are positive numbers A_1, A_2, B_1 and B_2, where

　　　　$A_1 ó A_2 = B_1 ó B_2 > 0$　and　$A_1 > B_1$.

(b)　Find the areas of any six squares satisfying the following conditions:

　　(1) the lengths of the sides of the six squares are **integers**; and

　　(2) the areas of the six squares are positive numbers A_1, A_2, A_3, B_1, B_2 and B_3, where

　　　　$A_1 ó A_2 = B_1 ó B_2 > 0$,　$A_2 ó A_3 = B_2 ó B_3 > 0$　and　$A_1 > B_1$.

~ 全卷完 End of paper ~

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